



# Analysis of Image Denoising using Wavelet Threshold Method

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**ABSTRACT:** We address the image denoising problem, where zero-mean white and homogeneous Gaussian additive noise is to be removed from a given image. Wavelet is a powerful tool for denoising a variety of signals. Here an image has been taken for denoising purpose with the help of HAAR Wavelet Transform. The noisy image is first decomposed into five levels to obtain different frequency bands. Then hard and soft thresholding method is used to remove the noisy coefficients by fixing the optimum thresholding value. In this paper, analysis of a gray image is carried out with additive white Gaussian Noise at zero mean that are applied on the image to produce noisy images. Selection of the denoising algorithm is application dependent. Hence, it is necessary to have knowledge about the noise present in the image so as to select the appropriate denoising algorithm. Here we put results of different approaches of wavelet based image denoising methods using several thresholding techniques such as hard and soft thresholding. A quantitative measure of comparison is provided by SNR (signal to noise ratio) and mean square error (MSE).

**KEYWORDS:** Image Enhancement, Wavelet Denoising, Hard Threshold, Soft Threshold, Additive white Gaussian Noise.

## I.INTRODUCTION

Images are often corrupted with noise during acquisition, transmission, and retrieval from storage media. Many dots can be spotted in a Photograph taken with a digital camera under low lighting conditions. Figure 1 is an example of such a Photograph. Appearance of dots is due to the real signals getting corrupted by noise (unwanted signals). On loss of reception, random black and white snow-like patterns can be seen on television screens, examples of noise picked up by the television. Noise corrupts both images and videos. The purpose of the denoising algorithm is to remove such noise.



Figure 1: Gray scale image of (a) Lena (b) a building affected with Gaussian Noise.



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Image denoising is needed because a noisy image is not pleasant to view. In addition, some fine details in the image may be confused with the noise or vice-versa. Many image-processing algorithms such as pattern recognition need a clean image to work effectively. Random and uncorrelated noise samples are not compressible. Such concerns underline the importance of denoising in image and video processing.

Images are affected by different types of noise. The work presented herein focuses on a zero mean additive white Gaussian noise (AWGN). Zero mean does not lose generality, as a non-zero mean can be subtracted to get to a zero mean model. In the case of noise being correlated with the signal, it can be de-correlated prior to using this method to mitigate it. The problem of denoising can be mathematically presented as follows,

$$Y = X + N$$

Where Y is the observed noisy image, X is the original image and N is the AWGN noise with variance  $\sigma^2$ . The objective is to estimate X given Y. A best estimate can be written as the conditional mean  $\hat{X} = E[X | Y]$ . The difficulty lies in determining the probability density function  $\rho(x|y)$ . The purpose of an image-denoising algorithm is to find a best estimate of X. Though many denoising algorithms have been published, there is scope for improvement.

## A. Introduction to the Wavelet Transform

A wave is usually defined as an oscillating function in time or space. Sinusoids are an example. Fourier analysis is a wave analysis. A wavelet is a “small wave” that has its energy concentrated in time and frequency. It provides a tool for the analysis of transient, non-stationary, and time-varying phenomena. It allows simultaneous time and frequency analysis with a flexible mathematical foundation while retaining the oscillating wave-like characteristic. A simple high level introduction to wavelets can be found in the articles by Daubechies et al. [1]-[2]. A signal or a function f(t) can often be better analyzed if it is expanded as

$$f(t) = \sum_k C_{j_0,k} \phi_{j_0,k}(t) + \sum_k \sum_{j>j_0} d_{j,k} \psi_{j,k}(t)$$

Where both j and k are integer indices.  $\psi_{j,k}(t)$  represents the wavelet expansion functions, and  $\phi_{j,k}(t)$  represents the scaling functions. They usually form an orthogonal basis. This expansion is termed as wavelet expansion. The term related to the scaling coefficients captures the average or coarse representation of the signal at the scale  $j_0$ . The term related to the wavelet coefficients captures the details in the signal from scale  $j_0$  onwards. The set of expansion coefficients ( $C_{j_0,k}$  and  $d_{j,k}$ ) is called the discrete wavelet transform (DWT) of f(t). The above expansion is termed as the inverse transform. Multi resolution analysis (MRA) and Quadrature mirror filters (QMF) are also important for evaluating the wavelet decomposition. In multi resolution formulation, a single event is decomposed into fine details. A quadrature mirror filter consists of two filters. One gives the average (low pass filter), while the other gives details (high pass filter). These filters are related to each other in such a way as to be able to perfectly reconstruct a signal from the decomposed components.

Decimation operations when removed result in more data samples in multi resolution domain. This redundancy helps in denoising. The two dimensional (2D) wavelet transform is an extension of the one dimensional (1D) wavelet transform. To obtain a 2D transform, the 1D transform is first applied across all the rows and then across all the columns at each decomposition level. Four sets of coefficients are generated at each decomposition level: LL as the average, LH as the details across the horizontal direction, HL as the details across the vertical direction and HH as the details across the diagonal direction.

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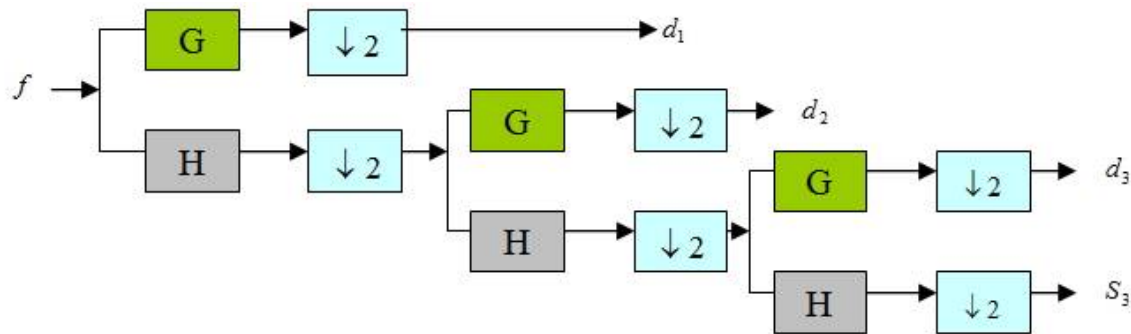


Figure: Wavelet filter bank for one level image decomposition

## II. RELATED WORKS

The VisuShrink method, introduced by Donoho [3], uses a threshold value  $T$  that is proportional to the standard deviation of the noise. It follows the hard thresholding rule. The threshold in VisuShrink is also referred to as universal threshold and it is defined as:

$$T = \sigma\sqrt{2 \log M}$$

Where  $\sigma$  is the noise variance present in the signal and  $M$  represents the signal size or number of samples. An estimate of the noise level  $\sigma$  is defined based on the median absolute deviation that is given by:

$$\sigma^2 = [(\text{median}|Y_{ij})/0.06745]^2$$

Where  $Y_{ij} \in HH_1$  subband

A threshold chosen based on Stein's Unbiased Risk Estimator (SURE) [4,5] is called as SureShrink. It is a combination of the universal threshold and the SURE threshold. This method specifies a threshold value  $t_j$  for each resolution level  $j$  in the wavelet transform which is referred to as level dependent thresholding. The goal of SureShrink is to minimize the mean squared error that is defined as:

$$MSE = \frac{1}{M^2} \sum_{i,j=1}^M (z(i,j) - s(i,j))^2$$

where  $z(x, y)$  is the estimate of the image and  $s(x, y)$  is the original image without noise and  $M \times M$  is the image size. The SureShrink suppresses the noise by thresholding the empirical wavelet coefficients. The SureShrink threshold  $t^*$  is defined as:

$$t^* = \min(t, \sigma\sqrt{2 \log M})$$

where  $t$  denotes the value that minimizes Stein's Unbiased Risk Estimator,  $\sigma$  is the noise variance, and  $M$  is the size of the image. SureShrink follows the soft thresholding rule. The thresholding employed in this case is adaptive, i.e., a threshold level is assigned to each dyadic resolution level by the principle of minimizing the Stein's Unbiased Risk Estimator for threshold estimates.

The BayesShrink [6] minimizes the Bayesian risk, and hence its name, BayesShrink. It uses soft thresholding and is subband-dependent, which means that the thresholding is done at each band of resolution in the wavelet decomposition. Like the SureShrink procedure, it is smoothness-adaptive. The Bayes threshold,  $t_B$ , is defined as:

$$t_B = \sigma^2 / \sigma_s$$

where  $\sigma^2$  is the noise variance and  $\sigma_s$  is the signal variance without noise. The noise variance  $\sigma^2$  is estimated from the subband  $HH_1$ .

The aim of local thresholding rules is to increase the estimation accuracy by utilizing information about neighboring wavelet coefficients. The block thresholding increases the estimation precision by utilizing the information



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about the neighbor wavelet coefficients. Recently, there has been a fair amount of research to select the threshold for image denoising from the noisy image using wavelet [7-8].

The choice of a threshold is an important point of interest. It plays a major role in noise removal of images because denoising most frequently produces smoothed images, reducing their sharpness. Generally, the choice should be taken to preserve the edges of the denoised image. Our proposed image denoising method is based on thresholding that not only removes noise but also preserves the edges. It is discussed in next section.

### III. METHODOLOGY

Before discussing our proposed work, we will review the wavelet transform in brief. Suppose  $X = \{X_{ij}, i = 1, 2, 3, \dots, M \text{ and } j = 1, 2, 3, \dots, N\}$  is an image of  $M \times N$  pixels, which is corrupted by independent and identically distributed (i.i.d) zero mean, white Gaussian noise  $n_{ij}$  with standard deviation  $\sigma_n$ . The noise signal can be denoted as  $n_{ij} \sim N(0, \sigma_n^2)$ . This noise may corrupt the signal in a transmission channel. The observed, noise contaminated, image is  $Y = \{Y_{ij}, i = 1, 2, 3, \dots, M \text{ and } j = 1, 2, 3, \dots, N\}$ . Therefore, the noised image can be expressed as:

$$Y_{ij} = X_{ij} + n_{ij}$$

The object of a de-noising process is to estimate image  $x$  from the noised image  $y$ , so that the Mean Square Error (MSE) to be minimum. Let  $W$  and  $W^{-1}$  denote the two dimensional DWT and its inverse respectively. Then, the original signal, its noised version and the noise have a matrix form in the transform domain that includes the subband coefficients.

$$X = W x, \quad Y = W y, \quad V = W n$$

Figure 3 shows the two level DWT of a 2-D image, which consists of the subbands  $LL_2$  (low frequency or approximation coefficients),  $HL_2$  (horizontal details),  $LH_2$  (vertical details),  $HH_2$  (diagonal details) and the first level details  $HL_1, LL_1, HH_1$ .

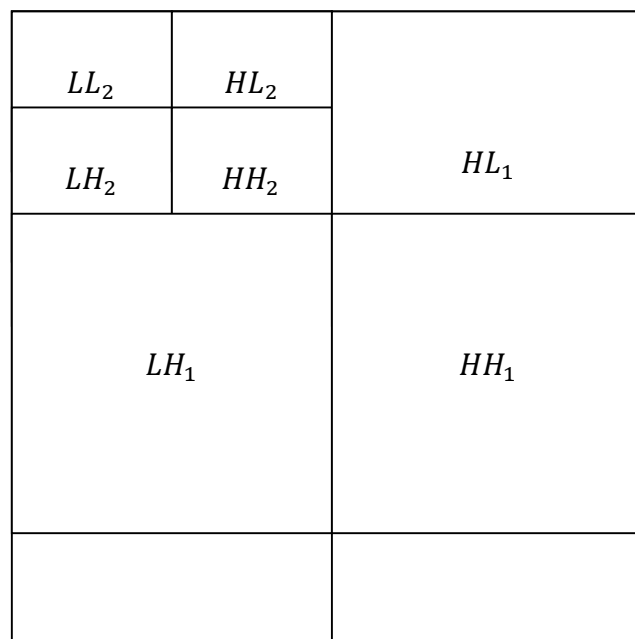


Figure 3: Two level Discrete Wavelet Transform of a 2-D image.

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Therefore first equation in the spatial domain, becomes in the transform domain as follows:

$$Y = X + V$$

where X, Y and V are the transform domains of the original image, its noised version and the noise respectively. The orthogonal property of the transform insures that the noise in the transform domain is also of Gaussian nature. The denoising algorithms, which are based on thresholding, suggest that each coefficient of every detail subband is compared to a threshold level and is either retained or killed if its magnitude is greater or less respectively. The approximation coefficients are not submitted in this process, since on one hand they carry the most important information about the image and on the other hand the noise mostly affects the high frequency subbands.

The type of the threshold is either hard or soft. Figure 4 indicates the two types of thresholding, which can be expressed analytically as follows.

## i. Hard Threshold

If the absolute value of a coefficient is less than a threshold, then it is assumed to be 0, otherwise it is unchanged. Mathematically it is

$$\hat{X} = \text{sign}(Y)(Y.*(\text{abs}(Y) > \lambda)),$$

Where Y represents the noisy coefficients,  $\lambda$  is the threshold,  $\hat{X}$  represents the estimated coefficients.

## ii. Soft Threshold

Hard thresholding is discontinuous. This causes ringing / Gibbs effect in the denoised image. To overcome this, Donoho [9] introduced the soft thresholding method. If the absolute value of a coefficient is less than a threshold  $\lambda$ , then is assumed to be 0, otherwise its value is shrunk by  $\lambda$ .

Mathematically it is

$$\hat{X} = \text{sign}(Y).*((\text{abs}(Y) > \lambda).*(\text{abs}(Y) - \lambda)),$$

This removes the discontinuity, but degrades all the other coefficients which tend to blur the image.

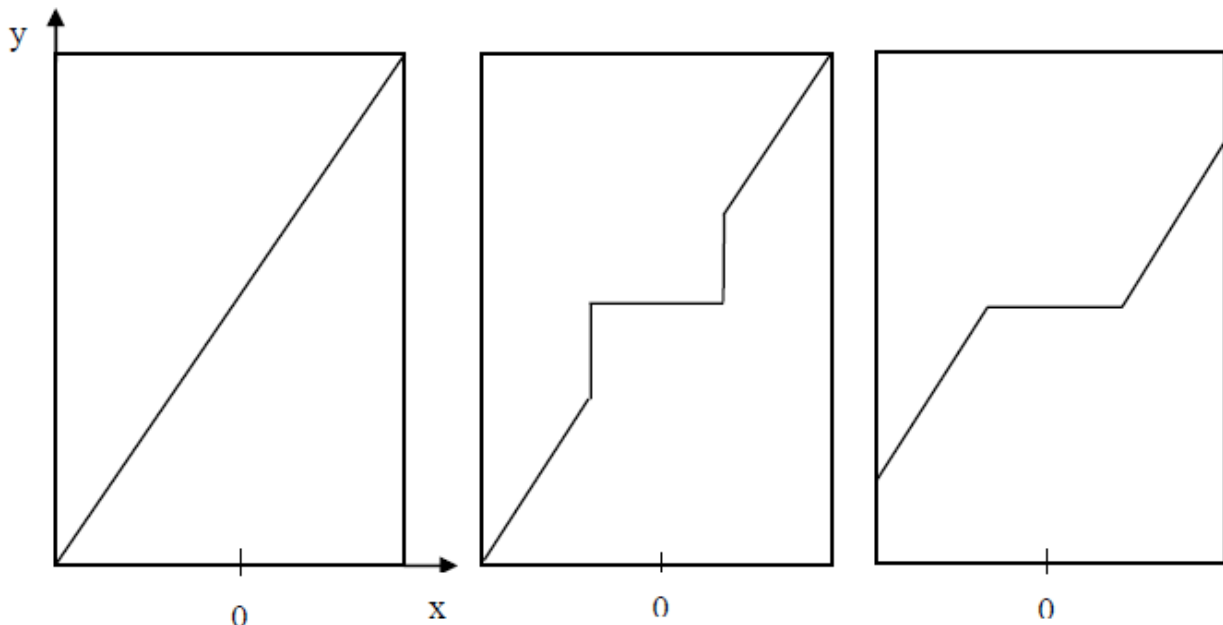


Figure 4. Threshold types: (a) Original signal; (b) Hard; (c) Soft

## Steps involved in this research work:

A gray scale image has been taken for denoising purpose in Wavelet Toolbox in MATLAB. Additive white Gaussian Noise is added in the original image of dimensions 832x640 in jpg format at zero mean and 0.02 variance. HAAR wavelet transform is applied on the noisy images of all the four different noises to decompose the noisy image up to five levels. A constant threshold value is taken at each level. Each coefficient of vertical, horizontal and diagonal



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details is thresholded using hard threshold and soft threshold. As a result of inverse wavelet transform, we get the noise free image within vertical, horizontal and diagonal details.

2. Calculation of the statistical parameters for residual image is obtained in the terms such as: (a) mean (b) median (c) mode (d) standard deviation (e) mean absolute deviation and (f) median absolute deviation.

3. Calculation of the performance parameters for analyzing noisy images and denoised images is performed in the terms such as: (a) RMSE (b) SNR (c) PSNR (d) MSE and (e) SSIM.

## IV. RESULT

**Table 1:** Mixed Noise Performance for different value of noise variance

Noise Variance	Mixed Noise Performance Parameter				
	SNR	PSNR	SSIM	MSE	RMSE
0.002	42.3810	48.0291	0.0095	17302.0191	131.5371
0.004	42.3822	48.0206	0.0092	17306.9425	131.5559
0.006	42.3831	48.0120	0.0091	17310.4605	131.5692
0.008	42.3802	48.0036	0.0090	17299.1187	131.5261
0.02	42.3764	47.9544	0.0093	17283.7735	131.4678

**Table 2:** PSNR value of the denoised image for different value of noise variance

Mixed Noise Variance	PSNR	
	Wavelet Denoising by hard threshold	Wavelet Denoising by soft threshold
0.002	48.0291	47.9173
0.004	48.0206	47.9086
0.006	48.0120	47.9000
0.008	48.0036	47.8915
0.02	47.9544	47.8410

**Table 2:** SSIM value of the denoised image for different value of noise variance

Mixed Noise Variance	SSIM	
	Wavelet Denoising by hard threshold	Wavelet Denoising by soft threshold
0.002	0.0095	0.0127
0.004	0.0092	0.0117
0.006	0.0091	0.0108
0.008	0.0090	0.0108
0.02	0.0093	0.0107

**Table 2:** MSE value of the denoised image for different value of noise variance

Mixed Noise Variance	MSE	
	Wavelet Denoising by hard threshold	Wavelet Denoising by soft threshold
0.002	17302.0191	16862.4883
0.004	17306.9425	16865.0000
0.006	17310.4605	16865.2909
0.008	17299.1187	16858.6068
0.02	17283.7735	16850.0820

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**Table 2:** SNR value of the denoised image for different value of noise variance

Mixed Noise Variance	SNR	
	Wavelet Denoising by hard threshold	Wavelet Denoising by soft threshold
0.002	42.3810	42.2692
0.004	42.3822	42.2699
0.006	42.3831	42.2699
0.008	42.3802	42.2682
0.02	42.3764	42.2660

**Table 2:** RMSE value of the denoised image for different value of noise variance

Mixed Noise Variance	RMSE	
	Wavelet Denoising by hard threshold	Wavelet Denoising by soft threshold
0.002	131.5371	129.8556
0.004	131.5559	129.8653
0.006	131.5692	129.8664
0.008	131.5261	129.8407
0.02	131.4678	129.8079



Figure 5: (a) Noisy image of lena



(b). Denoised image by hard threshold method



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(c). Denoised image by soft threshold method

## VI.CONCLUSION

It is necessary to image denoising processing to improve the quality of image. Much practical noise can be approximated as white noise with Gauss distribution, and removal of superposition of Gauss white noise has become an important direction in image denoising research. Since the concept of wavelet threshold has been proposed, for its optimal estimate in the Besov space, much attention has been paid on it and various algorithms based on it have been developed. Wavelet thresholding used for denoising is according to the adjustment of wavelet coefficients in the wavelet domain. Then we can clear the noise by setting threshold. In this paper, wavelet transform is used in the image denoising. Two typical images are used to verify the validity of the new algorithm, and the results show that the new algorithm can improve the signal to noise ratio compared with the traditional algorithm.

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