



Reconstruction Algorithm for Microwave Tomography Using Iterative Regularized Levenberg-Marquardt Method

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ABSTRACT: The theory and equations are developed for the scattering pattern of a dielectric cylinder of arbitrary cross section shape. The harmonic incident wave is assumed to have its electric vector parallel with the axis of the cylinder, and the field intensities are assumed to be independent of distance along the axis. Solutions are readily obtained for inhomogeneous cylinders when the permittivity is independent of distance along the cylinder axis. We treat the total field as an unknown function which is determined by solving a system of linear equations. In the case of the dielectric cylindrical shell of circular cross section, this technique yields results which agree accurately with the exact classical solution. Numerical solutions for the electromagnetic fields induced in an inhomogeneous biological medium are obtained using the method of moment's solution. To reconstruct images during the inverse problem we use Levenberg-Marquardt method.

KEYWORDS: Levenberg-Marquardt method.

I. INTRODUCTION

Microwave tomography techniques for biomedical applications are lagging behind imaging schemes based on X-ray, ultrasound application, nuclear magnetic resonance (NMR) and even electrical impedance tomography (EIT). During the past 20 years, immense researches are being carried out in microwave tomography to quantitatively reconstruct the complex permittivity distribution of the biological media. Spectral methods used for diffraction tomography have been investigated with application to microwave (Adams et al. 1982, Bolomey et al. 1982, 1990, Devaney 1983, Slaney et al. 1984, Rius et al. 1987). The advantage of such methods results from the existing fast numerical algorithm. However, the diffraction tomography suffers from the fact that it is marred in strongly inhomogeneous media where Born and Rytov approximations are not valid (Slaney et al. 1984, Bolomey et al. 1991). The other methods (Ney et al. 1984, Wang et al. 1989, Caorsi et al. 1990) based on moment method solutions are being explored rigorously, but the stability depends on the measurement accuracy due to ill-conditioning of the matrix.

II. FIELD INTEGRAL EQUATION

By employing a Green's function for half-space, it can be shown that the incident field at any point given in terms of the aperture field distribution $\mathbf{E}_a(x', y')$ by $\mathbf{E}^{\text{in}}(x, y) = -\frac{j}{2} \frac{\partial}{\partial x} \int \mathbf{E}_a(x', y') H_0^{(1)}[k\{(x-x')^2 + (y-y')^2\}^{1/2}] dy'$, where $2a$ denotes the length of the aperture. Numerical solutions for the electromagnetic fields induced in an inhomogeneous biological medium are obtained using the method of moment solution. A tensor integral equation approach usually produces an accuracy of high order, however the inverse problem of estimating the complex permittivity distribution would be rather involved. A relatively less accurate method described by Richmond (Richmond 1965) is used in our present analysis. This approach has been used in our present analysis because (i) The solutions are available in closed form and (ii) the inverse problem can be presented in a very convenient way. According to Richmond, if the biological medium under investigation is decomposed into n number of cells, the total field distributions \mathbf{E}_i ($i = 1, n$) within the cells when illuminated from an external source can be obtained from the relation $\mathbf{C} \mathbf{E} = \mathbf{E}_{\text{in}}$. Where \mathbf{C} is an $n \times n$ coefficient matrix, \mathbf{E} is an $n \times 1$ total field vector, \mathbf{E}_{in} is an $n \times 1$ incident field vector in vacuum. The elements C_{mp} 's are given by

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$$C_{mp} = 1 + (\epsilon_m - 1) (j/2) [\pi k a_m H_1^{(2)}(k a_m) - 2j] \quad \text{if } p=m$$

$$C_{mp} = (j \frac{\pi k a_p}{2}) (\epsilon_p - 1) J_1(k a_p) H_0^{(2)}(k \rho_{mp}) \quad \text{if } p \neq m$$

a_m, a_p are the radii of the equivalent circular cells having the same cross sectional area as cell m and p respectively : $K=(2\pi/\lambda_0)$ is the free space propagation constant, H_0 and H_1 are Hankel functions of order zero and one respectively, ρ_{mp} is the radial distance between cells m and p .

III. THEORETICAL MODEL

The theoretical model used to test our algorithm is shown in figure 1. It is high contrast square biological object $9.6 \text{ cm} \times 9.6 \text{ cm}$ consisting of muscle and bone having complex dielectric constants $50-j23$ and $8-j1.2$ respectively at a frequency of 1 GHz. The object is kept immersed in saline water having complex dielectric constant $76-j40$.

The target is illuminated with TE fields radiating from an open ended dielectric wave guide having sinusoidal field aperture field distribution. The transmitter is moved along four mutually orthogonal directions. For each of the transmitter positions along a particular transmitting plane, the received fields at eighteen locations in the other three orthogonal planes were measured theoretically described by Richmond (Richmond 1965) at a frequency 1 GHz. Therefore the measurement set contains 288 independent data. The rectangular model together with saline water region is divided into 324 equal square cells $0.6 \text{ cm} \times 0.6 \text{ cm}$.

FIGURE OF 2D MODEL

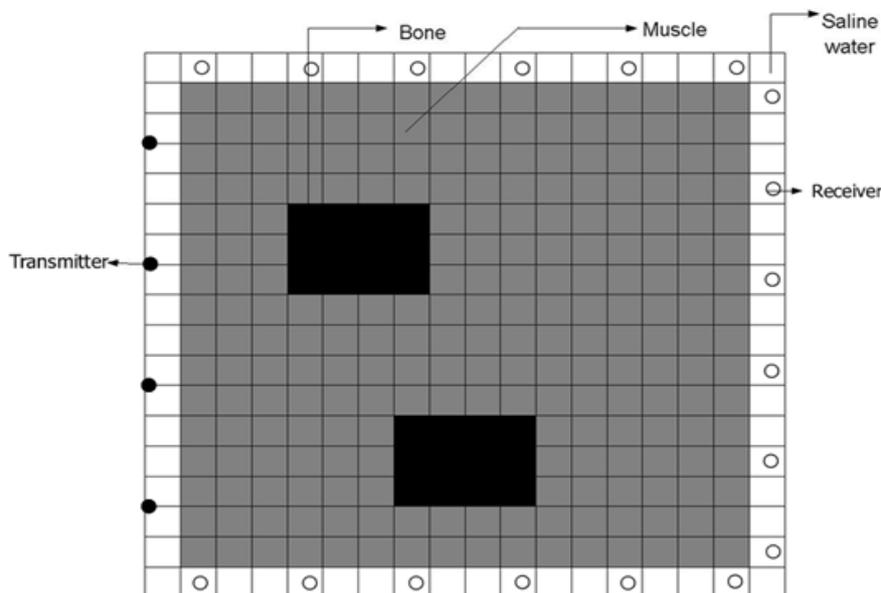


Fig1. Theoretical model, digitized cross-section, and different transmitter- receiver positions.

IV. INVERSE PROBLEM

we defined squared errors at receiver points as

$$\Phi = (E_r - E_0)^{\dagger} (E_r - E_0)$$

Where \dagger denotes the conjugate transpose and $E_0: C^m \rightarrow C^n$, the electric fields we measure at receiver points, $E_r : C^m \rightarrow C^n$, a function mapping the complex permittivity distribution with m degree of freedom into a set of n approximate electric field observation, and also



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$\epsilon \in C^m$, the complex permittivity distributions with m degrees of freedom.

We want to determine ϵ for which ϕ is minimum. We first differentiate ϕ with respect to ϵ' and set it equal to zero vector,

$$\phi' = [E_r'] / (E_r - E_0)$$

Where $[E_r']$ is an (n*m) Jacobian matrix and is defined by

$$[E_r']_n = (\delta[E_r] / \delta\epsilon_i)$$

Now expanding ϕ' in Taylor series and keeping only the linear terms, we have

$$\phi'(\epsilon + \Delta\epsilon) = \phi'(\epsilon) + \phi''(\epsilon) \cdot \Delta\epsilon$$

It has been stated in (Yorkey et. Al 1987) that the Hessian matrix ϕ'' can be written as

$$\phi'' = [E_r']^\dagger$$

Since the kronecker matrix product term is often negligible compared to $[E_r']^\dagger E_r'$

Hence $\Delta\epsilon$ can be written as

$$\begin{aligned} \Delta\epsilon &= -[\phi'']^{-1} \cdot \phi' \\ &= -[[E_r']^\dagger]^{-1} \cdot [E_r']^\dagger (E_r - E_0) \end{aligned}$$

Equation gives an iterative procedure to find ϵ . At ith iteration we find $\Delta\epsilon^i$ and update our permittivity values by

$$\epsilon^{i+1} = \epsilon^i + \Delta\epsilon^i$$

V. RECONSTRUCTION ALGORITHM

To apply the reconstruction algorithm, the biological medium was assumed to be a homogeneous one having complex dielectric constant 50-j23 i.e. it was assumed to be filled up with muscle. The receiver fields at different receiver location were computed for each transmitter position, using the described by Richmond the Jacobian matrix was computed. In the iterative equation, it was found that $[[E]^\dagger E]^{-1}$ was ill-conditioned, the condition number was found to be 12. The ill-conditioning makes its calculated inverse accurate for small dielectric constants updates only. We choose Levenberg Marquardt method to handle this ill-conditioning.

The Marquardt form is given by

$$(A + \lambda I) \Delta\epsilon = B$$

Where λ is a scalar, A is $[E_r']^\dagger E_r'$ and B is $[E_r']^\dagger (E_r - E_0)$.

The Marquardt method is as follows:

- (i) ϕ is computed.
- (ii) A value of λ is picked up ($\lambda = 0.0001$ in our case).
- (iii) Equation above is solved for $\Delta\epsilon$ and ϕ^{k+1} is evaluated
- (iv) If $\phi^{k+1} \geq \phi^k$, λ is increased by a factor 10 and control is transferred to step(iii)
- (v) If $\phi^{k+1} \leq \phi^k$, λ is decreased by a factor 10, the total solution is updated $\epsilon \leftarrow \epsilon + \Delta\epsilon$, and control is transferred to step (iii).

The only priori information we have used in our algorithm is that the real part of the complex dielectric constant cannot be negative and the imaginary part cannot be positive. Figure 2 shows the model in terms of the real and imaginary parts. Figure 3 shows the reconstructed images after 0.1,5,10,12, and 15 iterations. We define the Mean Estimation Error as:

$$\text{Mean Estimation Error} = \frac{1}{N} \sum \left| \frac{\epsilon_i - \epsilon_i^*}{\epsilon_i^*} \right| * 100$$

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VI.COMPUTATION OF THE JACOBIAN

Now we can write $E=C^{-1}E_m$

$$\begin{aligned} \text{Hence } [E']_{il} &= \delta[C^{-1}E_{in}]_i / \delta \epsilon_l \\ &= -C^{-1}(\delta C / \delta \epsilon_l) C^{-1} [E_{in}]_i \\ &= -C^{-1}(\delta C / \delta \epsilon_l) [E]_i \end{aligned}$$

To compute the Jacobian matrix $[E'_r]$, we notice that since the receiver is located at the center of some cells, hence E_r must be a subset of E . so we can conclude E'_r is also a subset of E' . In fact we had first determined E' , an $n \times n$ matrix, for each transmitter position. We retained only q number of row out of n rows, where q is the number of receivers for each transmitter position, so that the Jacobian matrix would be of order $(q \times n)$. We appended q rows of each transmitter position so that the Jacobian matrix would be of order $(qs \times n)$. Where s is the number of transmitter position. In our calculation, we have $q=18$, $s=16$ and $n=256$ so that the order of the Jacobian matrix was (288×256) .

VII. EXPERIMENTAL RESULTS

To apply the reconstruction algorithm, it was assumed that the biological medium is filled up with muscle only i.e. it was assumed to be homogeneous one having complex dielectric constant $50-j23$. The received fields at different receiver location were computed for each transmitter position. The only priori information we have used in our algorithm is that the real part of the complex dielectric constant can't be negative and the imaginary part can't be positive.

RECONSTRUCTED IMAGES FOR THE 2D MODEL.

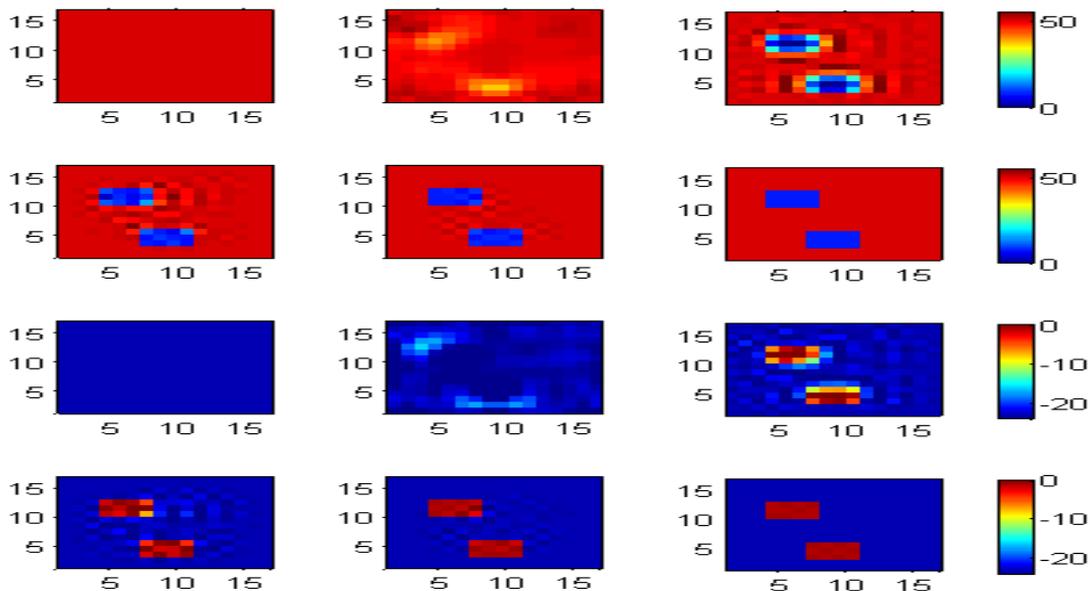


Fig2. Reconstructed model with real part and imaginary part



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Now plot the error vs. iteration ,we obtain the following graph. The iteration is stopped when the output error of the order of 10^{-4} .

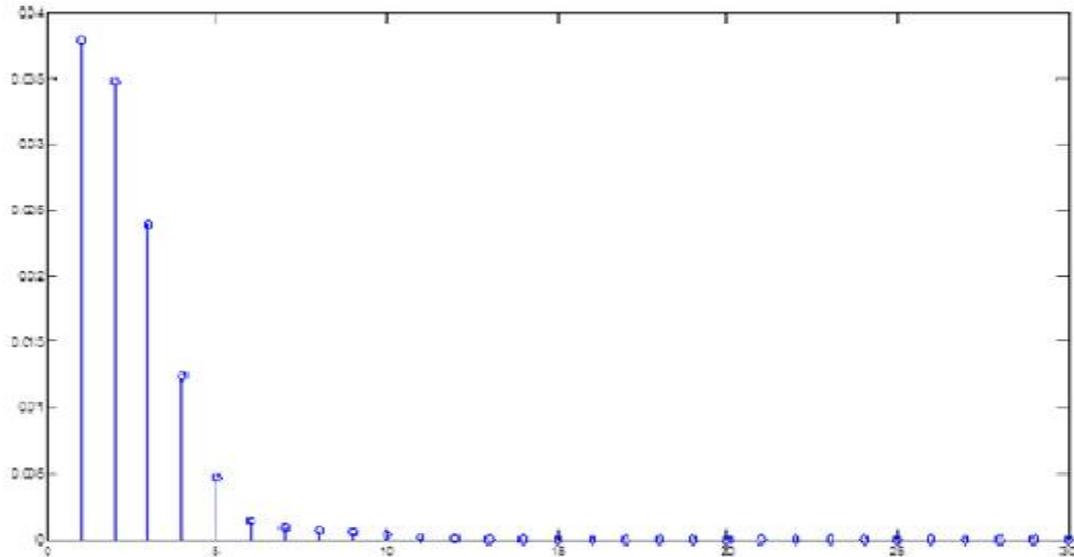


Fig3. This fig shows error vs. iteration

VIII.CONCLUSION

To apply there construction algorithm, it was initially assumed that the biological medium is filled up with muscle only. The received fields at different receiver locations were computed for each transmitter position. The only priori information we have used in our algorithmic that there all part of the complex dielectric constant cannot be negative and the imaginary part cannot be positive. The iteration is stopped when the 2-norm error output is of the order of 10^{-4} . In the first case, algorithms developed were in the ideal condition. i.e. Noise is absent. In this case reconstruction images were perfect. But when we introduced noise then reconstruction image is changes. We developed further algorithms (gauss-Newton algorithm) that shows that nosy reconstruction images becomes Similar that was obtained in ideal condition.

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