



Robust Stability Based Pid-Controller Design of D.C. Servo Plant with Time-Delay and Additive Uncertainty

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ABSTRACT: The area of control system engineering that mainly deals in obtaining the robustness of system in the existence of uncertainties is known as robust control. In this paper, a graphical design method is developed for obtaining the entire range of PID controller gains that stabilize a D.C. servo plant robustly with the existence of time delays and additive uncertainty employed. This method of design mainly works on the frequency response of the system, which can provide to decrease the complexities implicated in servo plant modeling. In fact in the real time processes, the time-delays and parametric uncertainties are more or less always present, that makes our controller design method crucial for process control. We have used this graphical method of design to find the robust stability of a DC servo plant model with a delay in communication and additive uncertainties. The results were found satisfactory and robust stability has achieved for the said model.

KEYWORDS: PID controller, robust control, DC servo plant, Time delay, Uncertainty, Stability.

I. INTRODUCTION

Presently the PID controllers are most effectively and widely used in the process control industries. The PID controller structure basically makes simple to control the process plant output. Graphical methods for designing PID controllers which operates effectively and optimally are simple, and these methods are very essential for process industries.

Robust control is concern with acquiring control systems which are indifferent to plant/model uncertainty or mismatch. In order to acquire the plant stability with respect to time uncertainties and time delays, wide research has been carried out in the designing methods of the controllers. This paper presents a design based on graphical method to get gains of the PID controller in order to achieve a robust stability, considering with parametric uncertainties and time delays for arbitrary order plants. Let us consider a model having an additive uncertainty, in order to acquire the whole set of uncertainty. The design methodology of H_∞ controller is considered to find the uncertain plant remains stable for the whole set of uncertainty.

Proposed design method lower down the complexities of plant modeling in the frequency domain application. This method of controller design considering with parametric uncertainties and time delays is then subjected to a DC servo plant model. Example of the DC servo plant shows that there will be assured robust and closed loop stability with the PID controller gains. Most of the initial works done in this field is mainly dedicated on finding that PID controllers which stabilizes a nominal plant model.

Hermite-Biehler theorem is generalized and used by Bhattacharyya and their colleagues in order to find out all stabilizing PID controllers considering with time delay for systems [6 to 8].

The authors of reference [9] Shows a novel method of controller design which did not depend on complicated mathematical derivations. They also extended their work by acquiring the complete area of PID controllers that meet with the requirements of certain phase and gain margin.

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Reference [10 to 13] shows some techniques in order to find out all those achievable PID controllers that satisfied complementary and weighted sensitivity, robust stability and performance constraints and those PID controllers that stabilized an arbitrary order system.

II. PROPORTIONAL - PLUS – INTEGRAL - PLUS DERIVATIVE CONTROLLERS

In 1939 PID controller was first to be placed in the market and till now in the process control, this controller is most broadly used. More than 90% of the controllers are PID controllers and its advanced version that are used in the process industries; this is an investigation which is carried out in Japan in 1989. These controllers are quite familiar because derivative action is sensitive to the noise measurement. The main tool of the PID controller is the “PID control” which is the method of feedback control. Figure (1) indicates the fundamental structure of conventionally used feedback control systems. Below figure represents the process of the plant which has to be controlled.

Set-point value (r) has to be followed by process output variable (y), this is the purpose of control. This purpose can be achieved if the manipulated variable (u) modify at the command of the controller. Let us take an example of a process plant, assume some liquid is heated in a heating tank by the burning of fuel gas for some desired temperature. Temperature of the liquid is y (i.e. output variable of the process plant) and the flue gas flow is u (i.e. manipulated variable). Any factor that influences the output variable of the process except manipulated variable is a disturbance. It is assumed that the manipulated variable is added by a single disturbance in the Figure (1). However in some cases, a major disturbance enters the process or there is a need for considering the multiple disturbances. $e = r - y$, this equation defines error ‘ e ’.
 u (the manipulated variable) determined by the controller $C(s)$ computational rule based on its input data is the error (e) in the given figure (1).

One more thing which has to be noticed in Figure (1) is that it is assumed that detector measures the process output variable (y), the controller input can be considered as exactly being equal to y , with adequate accurateness instantly, which is not clearly mentioned here.

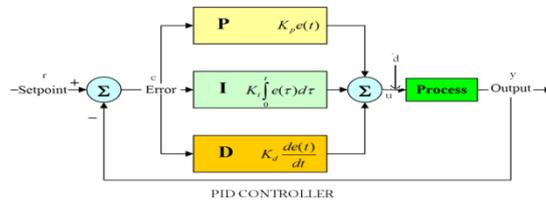


Fig.(1) PID Controller

Hence, the PID controller can be understood as a controller that takes into consideration: the past, the present and the future of the error. $G_C(s)$ known as gain or transfer function of the PID controller can be given by:

$$G_C(s) = K_p \left(1 + \frac{1}{sT_i} + T_d s \right) \dots\dots\dots (1)$$

$$G_C(s) = K_p + \frac{K_i}{s} + K_d s \dots\dots\dots (2)$$

Where,

- Controller proportional gain is indicated by K_p ,
- Controller integral gain is indicated by K_i , and
- Controller derivative gain is indicated by K_d .

The controllers can provide control action designed for specific process requirement by tuning the gains of these PID controllers [3].

III. THE CONCEPTS OF ROBUST CONTROL

Robust control is a field of the control system engineering that unambiguously deals by means of uncertainties in its approach to controller designing. The objective of the robust control methods is to achieve the robust stability and/or performance in the existence of uncertainties functions [1].

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Loop shaping technique is an important classical controller design method [1]. The classical feedback control methods were extended to a more renowned method based on shaping closed-loop transfer functions such as the weighted sensitivity function during the 1980's. These developments led to a more deep understanding of robust control concepts. An extensive research had been made during this period of time covering the number of techniques for modern robust control concepts and its uses to real-world systems [3].

IV. UNCERTAINTY MODEL

If differences in between the model of the system, used in designing a controller and of the actual system is not sensitive to a control system than such system is known as robust control system. These types of differences simply represent uncertainty in model or model/plant mismatch.

Furthermore, the key plan of the robust control is to verify whether the design specifications are met for the “worst-case” uncertainty [1]. The following approach to check robustness of the system.

1. Nominal stability of system should be checked.
2. Find out the Uncertainty Set: a mathematical representation of the system plant model uncertainty have to be determined.
3. Robust Stability (RS) Verification: Determine whether the system remains stable for all system plants in the uncertainty system.
4. Robust Performance (RP) Verification: If RS is satisfied; determine whether the performance specifications are met for all system plants in the uncertainty system.

A general block diagram representation of a one degree-of-freedom feedback control system is shown in figure (2), [1]. Where, r is the reference input, u is the controlled input to the plant, y is the actual plant output and d is the disturbance signal and n is the noise signal. The plant model, disturbance model and controller gains are represented by G , G_d , and K respectively.

The main objective of a control system is to make the output y must act in a desired manner by manipulating u such that the control error remains very small in spite of the disturbances present in the system. The system output can be denoted as:

$$Y = G(s)u + G(s)d \dots \dots \dots (3)$$

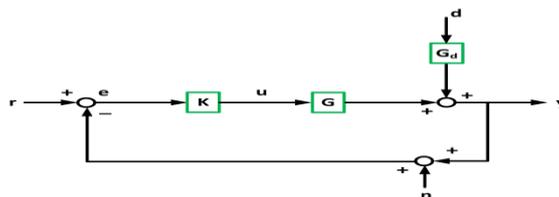


Fig.(2) Feedback control system with one degree-of-freedom [2].

V. TIME-DELAY SYSTEMS

It is to be found that most of the real-time systems have time-delay associated with them. There may be one or more of the following reasons of Time delay origination [2]:

1. System variables measurement.
2. Physical properties of the apparatus used in the system.
3. Transport delay in the signal transmission.

The effect of the time delay on a system depends on the size of the delay and system characteristics. Systems where the time delay plays a vital role are control, economic, political, biological, and environmental systems. A few examples include speed control of an engine, a cold rolling mill, spaceship control, unman-vehicle and hydraulic systems [2]. A block diagram representation of a cascade time delay system illustrated in figure (3).

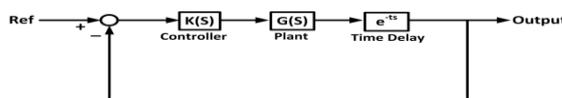


Fig.(3) Time delay system

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VI. UNCERTAINTY AND ROBUST STABILITY FOR SINGLE-INPUT -SINGLE-OUTPUT (SISO) PLANT SYSTEMS [16]

In order to design the control system, a robust control system always strives to remain insensitive towards the differences between the actual system and the model of the system.

In this paper, the primary objective is to determine the set of PID controllers that will guarantee robust stability for any arbitrary order SISO plant in the presence of time-delay and additive uncertainties.

As mentioned in [1], the origins of model uncertainty are as follows:

1. Parameters in the linear model that are approximately known
2. Parameters that vary due to nonlinearities or changes in the operating conditions
3. Measurement devices often have imperfections
4. At high frequencies the structure and model order is often not known
5. Controller implemented may differ from the one obtained by solving the synthesis problem

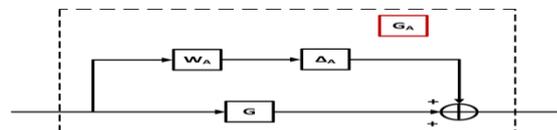


Fig.(4) Plant with additive uncertainty [1]

Based on the above criteria, the main classes of model uncertainties are as follows.

VII. PARAMETRIC UNCERTAINTY

In the Fig.4, $G_A(s)$ represented the perturbed plant which includes $\Delta_A(s)$, which is any stable transfer function such that $|\Delta_A(j\omega)| \leq 1, \forall \omega$. In the frequency domain we can represent these transfer functions as:

$$j\omega = \text{Re}(\omega) + j\text{Im}(\omega)$$

$$K(j\omega) = K_p + \frac{K_i}{j\omega} + K_d j\omega$$

$$W_A(j\omega) = A_A(\omega) + jB_A(\omega)$$

$$\| W_A(j\omega)K(j\omega)S(j\omega) \|_{\infty} \leq \gamma \dots \dots \dots (4)$$

If this condition is fulfilled and $\gamma=1$, then

$$S(j\omega) = \frac{1}{1+G_p(j\omega)K(j\omega)} \dots \dots \dots (5)$$

$$W_A(j\omega)K(j\omega)S(j\omega) = | W_A(j\omega)K(j\omega)S(j\omega) | e^{j\angle W_A(j\omega)K(j\omega)S(j\omega)}$$

$$W_A(j\omega)K(j\omega)S(j\omega) e^{j\theta_A} \leq \gamma \forall \omega$$

Or

$$\frac{W_A(j\omega)K(j\omega)}{1+G_p(j\omega)K(j\omega)} e^{j\theta_A} \leq \gamma \forall \omega \dots \dots \dots (6)$$

Where, $\theta_A = -\angle W_A(j\omega)K(j\omega)S(j\omega)$

$$P(\omega, \theta_A, \gamma) = 0$$

Where the system characteristic polynomial $P(\omega, \theta_A, \gamma)$ can be written as:

$$P(\omega, \theta_A, \gamma) = 1 + G_p(j\omega)K(j\omega) - \frac{1}{\gamma} \{ W_A(j\omega)K(j\omega) e^{j\theta_A} \}$$

$$e^{j\theta_A} = \cos \theta_A + j \sin \theta_A$$

$$P(\omega, \theta_A, \gamma) = 1 + \left((\text{Re } \omega + j\text{Im}(\omega)) \left(K_p + \frac{K_i}{j\omega} + K_d j\omega \right) \right) - \left(\frac{1}{\gamma} (A_A(\omega) + jB_A(\omega)) \left(K_p + \frac{K_i}{j\omega} + K_d j\omega \right) (\cos \theta_A + j \sin \theta_A) \right)$$

For $\gamma \rightarrow \infty$,

$$\mathbf{X}_{R_p} K_p + \mathbf{X}_{R_i} K_i + \mathbf{X}_{R_d} K_d = 0 \dots \dots \dots (A)$$

$$\mathbf{X}_{I_p} K_p + \mathbf{X}_{I_i} K_i + \mathbf{X}_{I_d} K_d = 0 \dots \dots \dots (B)$$

$$\mathbf{X}_{R_p} = -\omega \left(\text{Im}(\omega) + \frac{1}{\gamma} (A_A \sin \theta_A + B_A \cos \theta_A) \right) \dots \dots \dots (7)$$



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$$X_{R_i} = \left(\operatorname{Re}(\omega) + \frac{1}{\gamma} (A_A \cos \theta_A - B_A \sin \theta_A) \right) \dots\dots\dots(8)$$

$$X_{R_d} = -\omega^2 \left(\operatorname{Re}(\omega) + \frac{1}{\gamma} (A_A \cos \theta_A - B_A \sin \theta_A) \right) \dots\dots\dots(9)$$

$$X_{I_p} = \omega \left(\operatorname{Re}(\omega) + \frac{1}{\gamma} (A_A \cos \theta_A - B_A \sin \theta_A) \right) \dots\dots\dots(10)$$

$$X_{I_i} = \left(\operatorname{Im}(\omega) + \frac{1}{\gamma} (A_A \sin \theta_A + B_A \cos \theta_A) \right) \dots\dots\dots(11)$$

$$X_{I_d} = -\omega^2 \left(\operatorname{Im}(\omega) + \frac{1}{\gamma} (A_A \sin \theta_A + B_A \cos \theta_A) \right) \dots\dots\dots(12)$$

VIII. PID CONTROLLER DESIGN IN (K_p, K_d) PLANE FOR CONSTANT K_i

The transfer function model of the DC servo plant is represented as:

$$G(s) = \frac{53.27}{s(s+36.2)}$$

The boundary for $\mathbf{P}(\omega, \theta_A, \gamma) = \mathbf{0}$ for the (K_p, K_d) plane for a fixed value of $K_i = \bar{K}_i$ found using above equations (A) and (B), which can be rewritten as

$$\begin{bmatrix} X_{R_p} & X_{R_i} \\ X_{I_p} & X_{I_i} \end{bmatrix} \begin{bmatrix} K_p \\ K_d \end{bmatrix} = \begin{bmatrix} 0 - X_{R_i} \bar{K}_i \\ -\omega - K_{I_i} \bar{K}_i \end{bmatrix} \dots\dots\dots(13)$$

Solving equation (13) for all $\omega \neq \mathbf{0}$ and $\theta_A \in 0, 2\pi$, we obtain the controller gains as,

$$K_p(\omega, \theta_A, \gamma) = \frac{-\operatorname{Re}(\omega) - \frac{1}{\gamma} (A_A \cos \theta_A - B_A \sin \theta_A)}{X(\omega)}$$

$$K_d(\omega, \theta_A, \gamma) = \frac{\bar{K}_i}{\omega^2} + \frac{\operatorname{Im}(\omega) + \frac{1}{\gamma} (A_A \sin \theta_A + B_A \cos \theta_A)}{\omega X(\omega)}$$

Where,

$$X(\omega) = |G_p(j\omega)|^2 + \frac{1}{\gamma} |W_A(j\omega)|^2 + \frac{2}{\gamma} (\operatorname{Re}(\omega)(A_A \cos \theta_A - B_A \sin \theta_A) + \operatorname{Im}(\omega)(A_A \sin \theta_A + B_A \cos \theta_A))$$

Equation (13) can be rewritten as

$$\begin{bmatrix} -\omega\alpha(\omega) & -\omega^2\beta(\omega) \\ \omega\beta(\omega) & -\omega^2\alpha(\omega) \end{bmatrix} \begin{bmatrix} K_p \\ K_d \end{bmatrix} = \begin{bmatrix} 0 - X_{R_i} \bar{K}_i \\ -\omega - K_{I_i} \bar{K}_i \end{bmatrix}$$

Where,

$$\alpha(\omega) = \left(\operatorname{Im}(\omega) + \frac{1}{\gamma} (A_A(\omega) \sin \theta_A + B_A(\omega) \cos \theta_A) \right)$$

$$\beta(\omega) = \left(\operatorname{Re}(\omega) + \frac{1}{\gamma} (A_A(\omega) \cos \theta_A - B_A(\omega) \sin \theta_A) \right)$$

For $\omega = 0$ a solution may exist if $\bar{K}_i = 0$. Solving

$$\begin{bmatrix} -\alpha(0) & 0 \\ \beta(0) & 0 \end{bmatrix} \begin{bmatrix} K_p \\ K_d \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

We obtain $-\alpha(0)K_p = 0$ and $\beta(0)K_d = -1$.

For additive uncertainty weight the following form is selected:

$$w = \frac{M_h}{\left(s/\omega_{c_1} + 1 \right) \left(s/\omega_{c_2} + 1 \right)}$$

IX. RESULTS OF THE ROBUST CONTROLLER IN DIFFERENT PLANES:

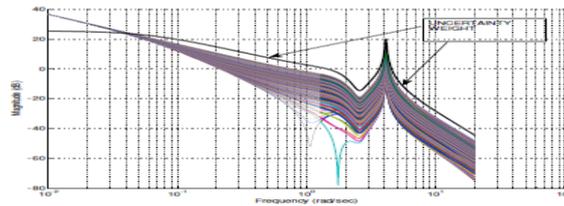


Fig.(5) Additive weight representation

The fig (5) shows the graph of additive uncertainties weight functions.

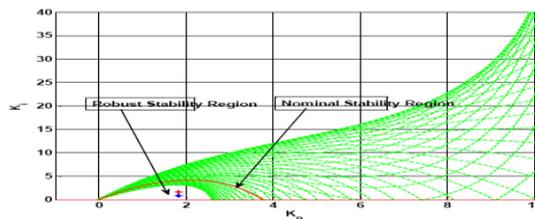


Fig.(6) Nominal stability boundary and robust stability region for K_p and K_i values

In fig. (6) the regions of nominal stability boundary and robust stability region for K_p and K_i values are illustrated.

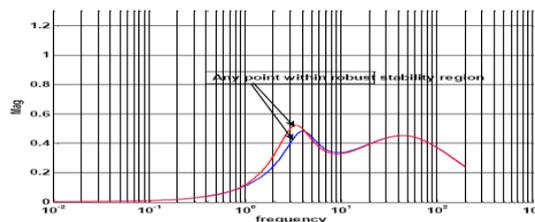


Fig.(7) Magnitude of $W_A(j\omega)K(j\omega)S(j\omega)$ for a point inside the robust stability region

Fig. (7) shows the frequency response of the robust controller with the plant. It also satisfies the condition of the robustness of the system.

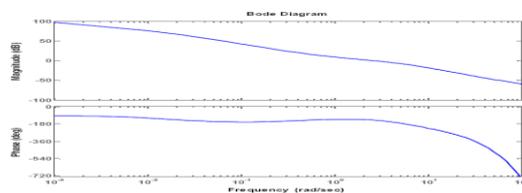


Fig.(8) Bode plot showing stable operation

In fig.(8) gives the information of the system's stable operation as per the bode plot criterion.

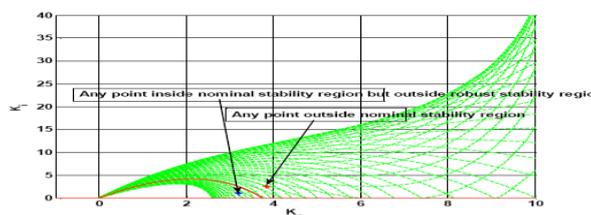


Fig.(9) Nominal stability boundary and robust stability region for K_p and K_i values

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The values of the K_p and K_i taken outside the nominal boundary as shown in fig. (9). For these values of K_p and K_i the system operations becomes unstable.

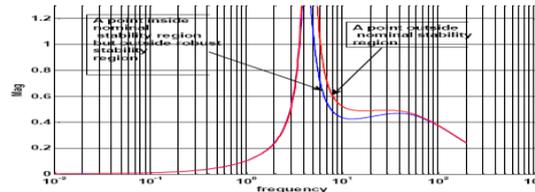


Fig.(10) Magnitude of $W_A(j\omega)K(j\omega)S(j\omega)$ for a point outside the robust stability region

Unstable Response of the system shown in fig. (10), where the criterion is not meet.

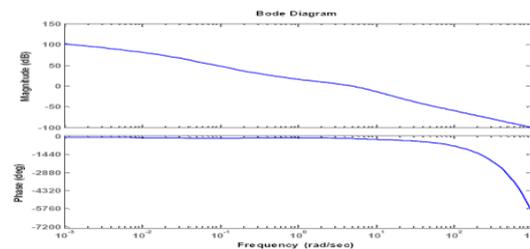


Fig.(11) Bode plot showing the unstable operation

The Bode plot for unstable operation of the system shown in fig. (11).

X. RESULTS DISCUSSION

The results are obtained shown in figures 5-11. In fig.6 additive uncertainty weight assign to the controller is shown fig.6 showing the points of operation within robust stability region and nominal stability region shown by red line. Within the points considered stability criterion i.e magnitude of $W_A(j\omega)K(j\omega)S(j\omega) < 1$ satisfied as shown in fig.7. For these points fig.8 showing the Bode plot with stable operation. Now we have considered two points, one within nominal stability region but outside of robust stability region and one point outside of nominal stability region as shown in fig.9. Result obtained in fig.10 does not satisfied the magnitude of stability criterion $W_A(j\omega)K(j\omega)S(j\omega) < 1$. Therefore, Bode plot in fig.11 showing the unstable operation. Similar results are obtained for (K_d, K_p) and (K_d, K_i) planes.

XI. CONCLUSION

A graphical design method for obtaining all PID controllers that satisfied a robust stability constraint for a DC servo plant with time delay and uncertainties has coded in MATLAB. Observing the results it can be concluded that the PID controllers selected from the robust stability regions in the (K_i, K_p) plane satisfy the robust stability constraint for the DC servo plant model. Using Bode plot's gain and phase margin Robust stability is founded. Similar results are found for (K_d, K_p) and (K_d, K_i) planes. It is concluded that all the Points within the robust stability region are always stable but all the points within the nominal stability region need not necessarily be always stable for different planes.

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