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# Studies of Load Flow on Radial Distribution Systems using Coupled Newton Raphson Method in Rectangular Coordinates

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**ABSTRACT:** Load flow is a very important and fundamental tool for the analysis of any power system and is used in the operational as well as planning stages. Load flow analysis aims at determination of system parameters like voltage, current, power factor power (real and reactive) flow at various points in the electric system under existing conditions of normal operation. Since the invention and widespread use of digital computers and many methods for solving the load flow problem have been developed. Planning the operation, looking out for a scope of expansion in future, all require a proper load flow study of the system.

**KEYWORDS:** NRFL, Load Flow, Bus Systems

## I. INTRODUCTION

Power flows studies, commonly referred to as *Load Flow, are the backbone of power* system analysis and design. They are necessary for planning, operation, economic scheduling and exchange of power between utilities; transient stability and contingency studies. The two primary considerations in the development of an effective engineering computer program are: The formulation of a mathematical description of the problem and the application of a numerical method for a solution.

Power flow analysis plays the major role during the operational stages of any system for its control and economic schedule, as well as during expansion and design stages. The purpose of any load flow analysis is to compute precise steady-state voltages and voltage angles of all buses in the network when the real and reactive power flows into every line and transformer under the assumption of known generation and load.

The objective of load flow study is to determine current, voltage, active power, reactive power etc. at various buses in power system operating under normal steady state or static condition. It requires planning of best operation and control of existing system and future expansion to keep pace with load growth. Load flow study helps in ascertaining the effect of new load, new generating stations, new lines and new interconnections before they are installed. It helps to minimize the system losses and also check is provided on system stability. Also it provides the proper pre-fault power system analysis to avoid system outage due to fault.

Successful operation of electrical systems requires that:-

- Generation must supply the demand (load) plus the losses.
- Bus voltage magnitudes must remain close to rated values.
- Generators must operate within specified real and reactive power limits.
- Transmission lines and transformers should not be overloaded for long periods.

Load flow studies give the information of voltage magnitude, phase angle magnitude, active power and reactive power.

## II. BUS CLASSIFICATION

The four variables Vi, δi, Pi and Qi are associated with the $i^{th}$ bus. Thus for the 'n'-bus system, there are a total of '4n' variables. As there are only '2n' equations available from the power-flow equation, out of these '4n' variables, '2n' quantities need to be specified & remaining '2n' quantities are solved from the '2n' load-flow equations. Also '2n' variables are to be specified in 'n' bus system, for each bus, two quantities need to be specified. For this purpose, the buses in a system are classified into three categories like load bus, generator bus and slack bus and each category contains two different quantities.

### A. For a n-bus, m-generator power system:

Classification of buses

| Type | Total no. of buses | Specified quantity | Solution quantity |
|---|---|---|---|
| PQ | n-m | $P_i, Q_i$ | $V_i, \delta_i$ |
| PV | m-1 | $P_i, V_i$ | $Q_i, \delta_i$ |
| Slack | 1 | $V_i, \delta_i$ | $P_i, Q_i$ |

**Table-1:** classification of bus system

## III. LOAD FLOW EQUATION

In rectangular co-ordinates, every complex quantity is expressed in terms of its real and imaginary parts. Hence, the basic Power Flow Equations are:

$$V_k = |V_k| e^{j\delta_i} = e_i + jf_i \text{ ----eqn1}$$

$$P_i - jQ_i = V_i^* I_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k \text{ ----eqn2}$$

$$Y_{ik} = G_{ik} - B_{ik} \text{ ----eqn3}$$

$$P_i - jQ_i = (e_i + jf_i)^* \sum_{k=1}^{n} (G_{ik} - jB_{ik})(e_i + jf_i)$$

$$=> P_i - jQ_i = (e_i - jf_i) \sum_{k=1}^{n} (G_{ik} - jB_{ik})(e_i + jf_i) \text{ ----eqn4}$$

$$P_i = \sum_{k=1}^{n} [e_i (G_{ik} e_k + B_{ik} f_k) + f_i (G_{ik} f_k - B_{ik} e_k)] \text{ ----eqn5}$$

$$Q_i = \sum_{k=1}^{n} [f_i (G_{ik} e_k + B_{ik} f_k) - e_i (G_{ik} f_k - B_{ik} e_k)] \text{ ----eqn6}$$

### A. Newton Raphson's Method:

Newton Raphson method is an iterative method which approximates the set of non-linear simultaneous equations using Taylor series expansion and the terms are limited to first order approximation.For solving solving a set of non-linear algebraic equation by means of basic NR algorithm, let there be 'n' equations in 'n' unknown variables $x_1, x_2, \ldots, x_n$ as given below

$$\left.\begin{array}{lcl} f_1(x_1, x_2, \cdots\cdots x_n) & = & b_1 \\ f_2(x_1, x_2, \cdots\cdots x_n) & = & b_2 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ f_n(x_1, x_2, \cdots\cdots x_n) & = & b_n \end{array}\right\} \text{ ------- eqn 7}$$

In equation (7), the quantities $b_1, b_2, \ldots, b_n$ as well as the functions $f_1, f_2, \ldots, f_n$ are known.
To solve equation (7), first we have to take an initial guess of the solution and let these initial guesses be denoted as, $x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}$.
Subsequently, first order Taylor's series expansion (neglecting the higher order terms) is carried out for these equation around the initial guess of solution.
Also let the vector of initial guess be denoted as

$X(0) = [x_1^{(0)}, x_2^{(0)}, \ldots, x_n^{(0)}]$

The application of Taylor's expansion on the equations of set (7) yields,

$$\left.\begin{array}{lcl} f_1\left(x_1^{(0)}, x_2^{(0)}, \cdots\cdots x_n^{(0)}\right) + \frac{\partial f_1}{\partial x_1}\Delta x_1 + \frac{\partial f_1}{\partial x_2}\Delta x_2 + \cdots\cdots + \frac{\partial f_1}{\partial x_n}\Delta x_n & = & b_1 \\ f_2\left(x_1^{(0)}, x_2^{(0)}, \cdots\cdots x_n^{(0)}\right) + \frac{\partial f_2}{\partial x_1}\Delta x_1 + \frac{\partial f_2}{\partial x_2}\Delta x_2 + \cdots\cdots + \frac{\partial f_2}{\partial x_n}\Delta x_n & = & b_2 \\ \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots \\ f_n\left(x_1^{(0)}, x_2^{(0)}, \cdots\cdots x_n^{(0)}\right) + \frac{\partial f_n}{\partial x_1}\Delta x_1 + \frac{\partial f_n}{\partial x_2}\Delta x_2 + \cdots\cdots + \frac{\partial f_n}{\partial x_n}\Delta x_n & = & b_n \end{array}\right\} \quad \text{------ eqn 8}$$

Equation (8) can be written as

$$\begin{bmatrix} f_1(\mathbf{x}^{(0)}) \\ f_2(\mathbf{x}^{(0)}) \\ \vdots \\ f_n(\mathbf{x}^{(0)}) \end{bmatrix} + \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix} \quad \text{-------- eqn 9}$$

Again equation (9) can be written as

$$\begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \vdots \\ \Delta x_n \end{bmatrix} = [\mathbf{J}]^{-1} \begin{bmatrix} b_1 - f_1(\mathbf{x}^{(0)}) \\ b_2 - f_2(\mathbf{x}^{(0)}) \\ \vdots \\ b_n - f_n(\mathbf{x}^{(0)}) \end{bmatrix} = [\mathbf{J}]^{-1} \begin{bmatrix} \Delta m_1 \\ \Delta m_2 \\ \vdots \\ \Delta m_n \end{bmatrix} \quad \text{-------- eqn 10}$$

Power Flow Analysis using Newton Raphson's Method in Rectangular Co-ordinates:

**B. NRLF (Rectangular) algorithm:**

**Step 1**: Assume flat start profile and denote the initial real and imaginary parts of the bus voltages as $e^{(0)}$ and $f^{(0)}$ respectively.

**Step 2:** Set iteration counter <iter> = 1.

**Step 3:** Compute the vectors $P_{cal}$ and $Q_{cal}$ with the vectors $e^{(k-1)}$ and $f^{(k-1)}$ thereby forming the mismatch vector M. Let this vector be represented as $\Delta M = [\Delta M_1, \Delta M_2, \ldots, \Delta M_{(2n-2)}]^T$.

**Step 4:** Compute:
error = max $(|\Delta M_1|, |\Delta M_2|, \ldots |\Delta M_{2n-2}|)$.

**Step 5**: If error <= (pre - specified tolerance), then the final solution vectors are $e^{(k-1)}$ and $f^{(k-1)}$ and print the results. Otherwise go to step 6.

**Step 6:** Evaluate the Jacobian matrix (J) with the vectors $e^{(k-1)}$ and $f^{(k-1)}$.

**Step 7:** Compute the correction vector X by solving equation (14).

**Step 8**: Update the solution vectors $e^{(k)} = e^{(k-1)} + \Delta e$ and $f^{(k)} = f^{(k-1)} + \Delta f$. Update iter = iter+1 and go back to step 3.

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ . \\ . \\ . \\ \Delta P_n \\ \Delta Q_2 \\ \Delta Q_3 \\ . \\ . \\ . \\ \Delta Q_n \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial e_2} & \frac{\partial P_2}{\partial e3} & . & . & . & \frac{\partial P_2}{\partial e_n} & \frac{\partial P_2}{\partial f_2} & \frac{\partial P_2}{\partial f_3} & . & . & . & \frac{\partial P_2}{\partial f_n} \\ \frac{\partial P_3}{\partial e_2} & \frac{\partial P_3}{\partial e_3} & . & . & . & \frac{\partial P_3}{\partial e_n} & \frac{\partial P_3}{\partial f_2} & \frac{\partial P_3}{\partial f_3} & . & . & . & \frac{\partial P_3}{\partial f_n} \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ \frac{\partial P_n}{\partial e_2} & \frac{\partial P_n}{\partial e_3} & . & . & . & \frac{\partial P_n}{\partial e_n} & \frac{\partial P_n}{\partial f_2} & \frac{\partial P_n}{\partial f_3} & . & . & . & \frac{\partial P_n}{\partial f_n} \\ \frac{\partial Q_2}{\partial e_2} & \frac{\partial Q_2}{\partial e_3} & . & . & . & \frac{\partial Q_2}{\partial e_n} & \frac{\partial Q_2}{\partial f_2} & \frac{\partial Q_2}{\partial f_3} & . & . & . & \frac{\partial Q_2}{\partial f_n} \\ \frac{\partial Q_3}{\partial e_2} & \frac{\partial Q_3}{\partial e_3} & . & . & . & \frac{\partial Q_3}{\partial e_n} & \frac{\partial Q_3}{\partial f_2} & \frac{\partial Q_3}{\partial f_3} & . & . & . & \frac{\partial Q_3}{\partial f_n} \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ \frac{\partial Q_n}{\partial e_2} & \frac{\partial Q_n}{\partial e_3} & . & . & . & \frac{\partial Q_n}{\partial e_n} & \frac{\partial Q_n}{\partial f_2} & \frac{\partial Q_n}{\partial f_3} & . & . & . & \frac{\partial Q_n}{\partial f_n} \end{bmatrix} . \begin{bmatrix} \Delta e_2 \\ \Delta e_3 \\ . \\ . \\ . \\ \Delta e_n \\ \Delta f_2 \\ \Delta f_3 \\ . \\ . \\ . \\ \Delta f_n \end{bmatrix} ----eqn11$$

Equation 11 can be re-written as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta e \\ \Delta f \end{bmatrix} ----eqn12$$

$$or, [\Delta M] = [J][\Delta X] ----eqn13$$

$$or, [\Delta X] = [J]^{-1}[\Delta M] ----eqn14$$

In step 6, the Jacobian matrix needs to be evaluated at each iteration. These purposes, the analytical expressions of the elements of the Jacobian matrix are needed.

**C. Derivation of Jacobian matrix elements for NRLF (rectangular) technique for Radial Distribution System:**

Elements of Jacobian Matrix can be derived from equation 5 and 6.

The off-diagonal elements of J1 are:

$$\frac{\partial P_i}{\partial e_k} = e_i G_{ik} - f_i B_{ik}, i \neq k ----eqn15$$

The diagonal elements of J1 are:

$$\frac{\partial P_i}{\partial e_i} = 2e_i G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} (e_k G_{ik} + f_k B_{ik}) ----eqn16$$

The off-diagonal elements of J2 are:

$$\frac{\partial P_i}{\partial f_k} = e_i B_{ik} + f_i G_{ik}, i \neq k ----eqn17$$

The diagonal elements of J2 are:

$$\frac{\partial P_i}{\partial e_i} = 2 f_i G_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} (f_k G_{ik} - e_k B_{ik}) ----eqn18$$

The off-diagonal elements of J3 are:

$$\frac{\partial Q_i}{\partial e_k} = e_i B_{ik} - f_i G_{ik}, i \neq k ----eqn19$$

The diagonal elements of J3 are:

$$\frac{\partial Q_i}{\partial e_i} = 2e_i B_{ii} - \sum_{\substack{k=1 \\ k \neq i}}^{n} (f_k G_{ik} - e_k B_{ik}) ----eqn20$$

The off-diagonal elements &diagonal elements of J4 are:

$$\frac{\partial Q_i}{\partial f_k} = -e_i G_{ik} + f_k B_{ik}, i \neq k ----eqn21$$

$$\frac{\partial Q_i}{\partial f_i} = 2 f_i B_{ii} + \sum_{\substack{k=1 \\ k \neq i}}^{n} (e_k G_{ik} + f_k B_{ik}) ----eqn22$$

$$\Delta P_i = P_{sch} - P_i^{(0)}$$
$$\Delta Q_i = Q_{sch} - Q_i^{(0)}$$

Then the mismatch vector $[\Delta P\ \Delta Q]^T$ can be calculated as follow
The superscript zero means the value calculated corresponding to initial guess, i.e., zeroth iteration.
After calculating the Jacobian Matrix and mismatch vector, the correction vector $[\Delta e\ \Delta f]^T$ can be calculated by using gauss elimination with sparsity technique.

The next better solution will be:
$e^{(k)} = e^{(k-1)} + \Delta e$ and $f^{(k)} = f^{(k-1)} + \Delta f$.
This process is continued till the magnitude of the largest element in the mismatch vector is less than the pre-specified tolerance.

## IV. DATA FOR 10 BUS SYSTEMS

| Bus No | $P_L$ (KW) | $Q_L$ (KVAR) | Sl. No | From Bus | To Bus | Line Resistance | Line Reactance |
|---|---|---|---|---|---|---|---|
| 1 | 0 | 0 | 1 | 1 | 2 | 0.1233 | 0.4127 |
| 2 | 1840 | 460 | 2 | 2 | 3 | 0.014 | 0.6051 |
| 3 | 980 | 340 | 3 | 3 | 4 | 0.7463 | 1.205 |
| 4 | 1790 | 446 | 4 | 4 | 5 | 0.6984 | 0.6084 |
| 5 | 1598 | 1840 | 5 | 5 | 6 | 1.9831 | 1.7276 |
| 6 | 1610 | 600 | 6 | 6 | 7 | 0.9053 | 0.7886 |
| 7 | 780 | 110 | 7 | 7 | 8 | 2.0552 | 1.164 |
| 8 | 1150 | 60 | 8 | 8 | 9 | 4.7953 | 2.716 |
| 9 | 980 | 130 | 9 | 9 | 10 | 5.3434 | 3.0264 |
| 10 | 1640 | 200 | | | | | |

**Table 2:** 10 bus system

## V. RESULTS

The results by using the methods explained above for the 10 Bus distribution systems are given in the following tables.

| Bus No. | e p.u | f p.u | V Mag p.u | d p.u |
|---|---|---|---|---|
| 1 | 1.0000 | 0 | 1.0000 | 0 |
| 2 | 0.9927 | -0.0090 | 0.9927 | -0.0091 |
| 3 | 0.9868 | -0.0219 | 0.9872 | -0.0220 |
| 4 | 0.9625 | -0.0393 | 0.9633 | -0.0406 |
| 5 | 0.9471 | -0.0439 | 0.9480 | -0.0462 |
| 6 | 0.9150 | -0.0594 | 0.9171 | -0.0648 |
| 7 | 0.9045 | -0.0653 | 0.9071 | -0.0721 |
| 8 | 0.8860 | -0.0715 | 0.8890 | -0.0805 |
| 9 | 0.8548 | -0.0809 | 0.8586 | -0.0942 |
| 10 | 0.8328 | -0.0875 | 0.8375 | -0.1046 |

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| Line No. | From Bus | To Bus | P Loss (KW) | Q Loss (KVar) |
|---|---|---|---|---|
| 1 | 1 | 2 | 46.6725 | 156.2192 |
| 2 | 2 | 3 | 3.9762 | 171.8585 |
| 3 | 3 | 4 | 177.2161 | 286.1386 |
| 4 | 4 | 5 | 114.3988 | 99.6562 |
| 5 | 5 | 6 | 190.2196 | 165.7117 |
| 6 | 6 | 7 | 47.7753 | 41.6166 |
| 7 | 7 | 8 | 75.7417 | 42.8976 |
| 8 | 8 | 9 | 88.4646 | 50.1052 |
| 9 | 9 | 10 | 39.3080 | 22.2633 |
| Total No. of Iterations: 4<br>Total Real Power Loss (in KWatts): 783.7730<br>Total Reactive Power Loss (in KVars): 1.0364e+003 | | | | |

Table 3: active power loss and reactive power loss between different buses

## VI. DISCUSSION

In the Newton Raphson Load Flow technique, the convergence characteristics is quadratic and the error in the current iteration is square of the error in previous iteration

The Newton-Raphson (NR) approach of Load Flow Solution is comparatively better than the other load flow techniques. It can solve cases that lead to divergence, but the NR method too has some limitations.

As the number of equations are reduced and the Jacobian is approximated by a constant triangular matrix, it is significantly faster than the implicit Gauss Seidel method or the traditional NR method.

The Jacobian is triangular and each function evaluation involves updating each bus voltage and current. So, the computation in each iteration is proportional to n, making it suitable for very large radial distribution systems.

Also the execution time is considerable saved.

## VII. CONCLUSION

Newton-Raphson (NR) approach of Load Flow Solution is better in comparison to the other load flow techniques. It is faster than the implicit of Gauss Seidel method or the traditional NR method. Time of execution is saved in this method. It is suitable for very large radial distribution systems.

## REFERENCES

[1] F. Zhang and C. S. Cheng, "A Modified Newton Method for Radial Distribution System Power Flow Analysis", IEEE Transactions on Power Systems,vol.12, no.1, pp.389-397, February 1997.

[2] Das D, Kothari D.P and Kalam A, "Simple and Efficient Method for Load Flow Solution of Radial Distribution Networks", Electrical Power & Energy Systems, Vol.17, no:5, pp.335-346, 1995

[3] Das D and Ghosh S, "Method for Load Flow Solution of Radial Distribution Networks", IEEE Proc. Gen., Trans., and Dist., Vol. 146, (6), pp. 641-648, 1999.

[4] Elgerd, O. I. 1982. *Electric Energy Systems Theory—An Introduction, 2nd ed. New York: McGraw*-Hill.

[5] Grainger, J. J., and W. D. Stevenson. 1994. *Power System Analysis. New York: McGraw-Hill.*