ISSN (Print) : 2320 – 3765
ISSN (Online): 2278 – 8875

International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering

(A High Impact Factor, Monthly, Peer Reviewed Journal)

Website: www.ijareeie.com

Vol. 8, Issue 6, June 2019

Comparison of Aircraft Pitch Control using Pole Placement and LQR

Aysha S Hameed [1], Al Bilal.N[2], Mukund A R[3] , Ansha.S [4]

Assistant Professor, Dept. of EEE, ACE College of Engineering, Trivandrum, Kerala India[1]

UG Student, Dept. of EEE, ACE College of Engineering, Trivandrum, Kerala, India[2]

UG Student, Dept. of EEE, ACE College of Engineering, Trivandrum, Kerala, India[3]

UG Student, Dept. of EEE, ACE College of Engineering, Trivandrum, Kerala, India[4]

ABSTRACT: The paper compares pitch control of a fighter aircraft in the longitudinal plane using pole placement method and Linear Quadratic Regulator method. The longitudinal dynamics of the aircraft is linearized to obtain the state space model with elevator deflection as control input and choosing angle of attack, pitch angle as well as pitch rate as state variables. In order to achieve the desired performance specifications, a controller is designed using pole placement method and compared with LQR method and the step response as well as stability of the system is analyzed. Simulation results validate that the required performance specifications are satisfied and LQR method gives better performance compared to pole placement and the desired pitch angle is attained with minimal overshoot and settling time.

KEYWORDS: Linear Quadratic Regulator (LQR), state space, linearization, overshoot.

I. INTRODUCTION

The development of modern control techniques has led to better control strategies in aviation, thereby improving safety and reliability. Pole placement method is used in the design of different control systems. This method employed in feedback control system theory distinguishes it from classical approach is that not only dominant poles but all poles can be laid at desired locations [7],[8]. The system is controlled by placing closed loop poles at desired locations and analyse the performance. Linear Quadratic Regulator (LQR) control is used to generate an optimum gain which balances the system error and the control effort based on a cost function which defines the relative importance of minimizing errors and minimizing control effort [9]. This paper focuses on the comparison of pole placement and LQR control of AFTI/F16 fighter aircraft to achieve a set of desired performance specifications.

Several control techniques both linear [5][6] and non-linear have been implemented for longitudinal control of general aviation as well as fighter aircrafts [1],[2],[4]. Longitudinal autopilot for a Jet Transport Aircraft has been implemented in [1] by analysis on time response and a settling time of 20 seconds has been achieved using PID control. Longitudinal control of four different types of aircraft and the performance analysis using root locus and bode plot has been done in [2]. The response obtained for fighter aircraft is found to be oscillatory with more settling time compared to the pole placement. LQR control of a general aviation aircraft implemented in [2] gives good response with respect to settling time but the method has not been applied to fighter aircraft for which time domain specifications in terms of settling time and non-oscillatory response is critical. Aircraft dynamic parameters and variables are taken from [10].

Section 2 describes the Vehicle Model, Section 3 deals with the Equations of Motion and State Space Model, Section 4 focuses on Problem Definition, Section 5 explains Pole Placement Design, Section 6 explains LQR method, Section 7 describes design of pre-compensator gain, Section 8 explains the Pole Placement Design Results and Section 9 explains LQR Design Results.

DOI:10.15662/IJAREEIE.2019.E80606604

II. SYSTEM MODEL

The U.S air force employed the fighter aircraft AFTI F-16 (Fig.1) for more than 20 years in flight control, electronic targeting and cockpit design. The Advanced Fighter Technology Integration (AFTI) F-16 has successfully completed more than 700 flights and has been tested in 10 different research programs. AFTI/F-16 has facilitated pilots to maneuver faster and lower while attacking targets. The vehicle control surfaces are Ailerons, Rudder and Elevator and the forces acting on the aircraft under equilibrium are Lift, Thrust ,Drag &Weight.

Fig 1. AFTI F-16 aircraft.

III. EQUATIONS OF MOTION AND STATE SPACE MODEL

The three longitudinal equations of motion comprises of the forces along the X, moment about Y, and force along Z axes are defined by (1), (2) and (3).

$$X - mgsin\theta = m(\dot{u} - rv + qw) \tag{1}$$

$$M = I_y\dot{q} + I_{xz}(p^2 - r^2) + rp(I_x - I_z) \tag{2}$$

$$Z + mgcos\theta cos\emptyset = m(\dot{w} + pv - qu) \tag{3}$$

The equations of motion can be linearized using small disturbance theory and by applying dynamic derivatives of AFTI/F-16 aircraft [9], equations (4), (5) and (6) are formulated.

$$\dot{\theta} = q \tag{4}$$

$$\dot{\alpha} = -1.1714 * 10^{-13}\theta - 0.76788\alpha + 0.9396q - 0.0016222 \tag{5}$$

$$\dot{q} = -2.251\alpha - 1.0462q - 0.16416\delta_e \tag{6}$$

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1.1714 * 10^{-13} & -0.76788 & 0.9396 \\ 0 & -2.251 & -1.0462 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ q \end{bmatrix} + \begin{bmatrix} 0 \\ -0.0016222 \\ -0.16416 \end{bmatrix} \delta_e$$

$$\theta = [1 \quad 0 \quad 0] \begin{bmatrix} \theta \\ \alpha \\ q \end{bmatrix} + [0]\delta_e \tag{7}$$

The control input is the elevator deflection, the state variables are chosen as pitch angle, angle of attack α and pitch rate, q.

IV. PROBLEM DEFINITION

The main objective is to design a controller for the AFTI/F16 fighter aircraft to satisfy a performance specification of 10% overshoot with a settling time less than 0.5 second. The transfer function of the system shown in (7) is given by

$$\frac{\theta(s)}{\delta_e(s)} = \frac{-0.1646s-0.1244}{s^3+1.814s^2+2.918s-2.637*10^{-13}} \qquad (8)$$

The system poles are located at $s = -0.907 \pm 1.4476i$ and $s = 0$. The root locus of the open loop system is shown in Fig. 2.

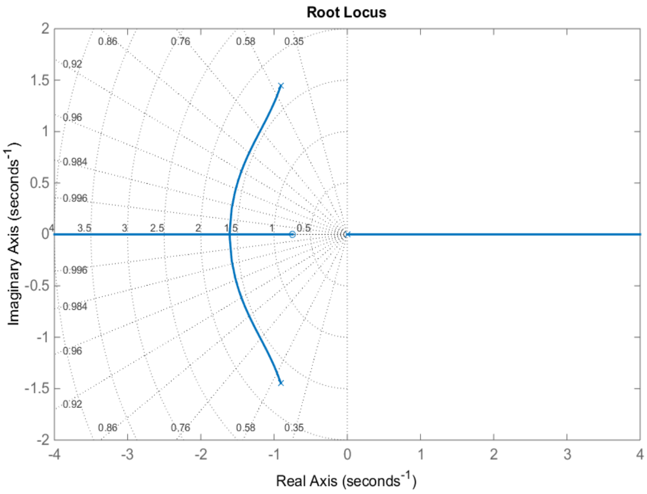

Fig 2. Root locus of the open loop system

The step response of the open loop system to a step input of 0.1radian is as shown in Fig.3. It is evident from the figure that the system is unable to track the step input. Hence, state feedback can be applied so that output can track the desired reference command.

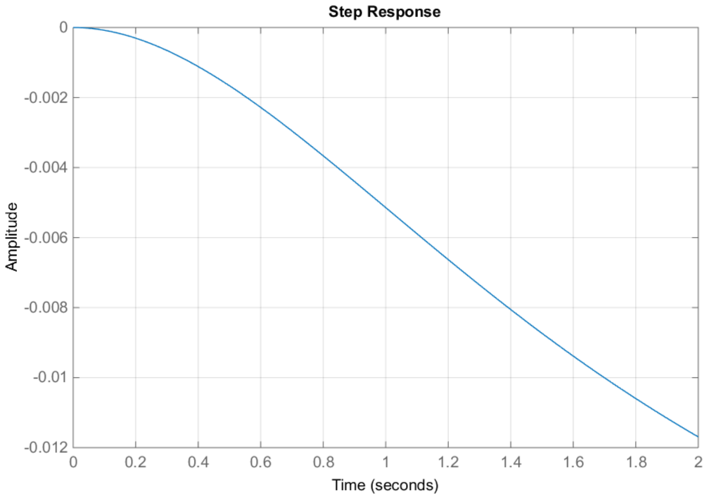

Fig 3. Step response of the open loop system

The open loop plant with state feedback is shown in Fig.4. The state feedback gain matrix K can be designed using pole/placement LQR technique.

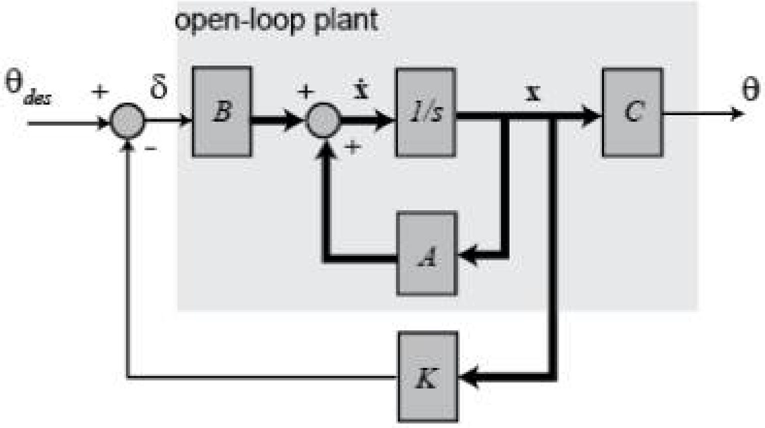

Fig 4. Plant with state feedback

V. POLE PLACEMENT DESIGN

Pole placement method is a controller design method in which the places of the closed loop poles are determined on the complex plane by setting a controller gain K. Placing the poles is ideal because the location of the poles corresponds directly to the Eigen values of the system, which control the characteristics and response of the system. The system must be controllable in order to implement this technique.

Poles describe the behavior of linear dynamical systems. Through use of feedback it can be attempted to change the behavior in a way that is more favorable .Determination of closed loop poles can be done based on the system desired transient response specifications and can be placed at desired locations using Pole Placement method through the design of controller gain K.

The state space model of the system is given by:

$$\dot{x} = Ax + Bu \quad (9)$$

Through use of full state feedback, control input is given by:

$$u = -Kx \quad (10)$$

with which the closed loop system matrix gives

$$A_c = A - BK. \quad (11)$$

A_c is a function of the controller gain K which can be used to place the eigen values at desired locations.

VI. LINEAR QUADRATIC REGULATOR DESIGN

Linear Quadratic Controller (LQR) is a linear control technique aimed at optimization of a quadratic objective or cost function. It is one of the most powerful controllers which is used for achieving the desired response. LQR controller parameters can be chosen to make the output of the system track the desired reference input, while driving the other system states to zero, satisfying the desired performance specifications.

In order to achieve the desired output pitch angle and drive the other system states to zero, the feedback controller gain ‘K’ needs to be evaluated. The conventional method of finding K is to determine the feedback gain by placing the closed loop poles at desired locations using pole placement. However, with respect to the change in desired poles, the controller gain K needs to be re-computed in this approach.

LQR design is based on optimization which selects the best solution considered as the most cost effective or higher achievable performance among so many alternative solutions. In the optimization process, desired factors are maximized and undesired factors are minimized.

Optimization is done by adjusting the cost functions of states and control input which are specified as weights Q and R respectively, Q is an n x n symmetric positive semi definite matrix and R is a n x n symmetric positive definite matrix. The two weights Q and R are varied with respect to the priority level of states or control input of the specified problem and thereby optimal cost function J is obtained as in (12). The first term minimizes the deviation of the states from the desired values and the second term minimizes the required control effort.

$$J = \int_0^T [x^T Q x + u^T R u] dt \tag{12}$$

After determining the cost function, the next step in LQR is the solution of algebraic Riccati equation given by (13). From the solution of Riccati equation an optimal controller gain K is formulated as in (14).

$$A^T P + PA - PBR^{-1}B^T + Q = 0 \tag{13}$$

$$K = R^{-1}B^T \mathrm{P} \tag{14}$$

The feedback control law is hence given by (15)

$$u = -Kx \tag{15}$$

VII. DESIGN OF PRE-COMPENSATOR GAIN G_N

A Pre-compensator gain G_N is used to scale the reference input so that the output can track the input in steady state. The step response (without using Pre-compensator gain) as shown in Fig. 6 it is evident from the figure that output is unable to track the desired response. The block diagram using Pre-Compensator shown in Fig. 7. The Gain G_N is to be used as shown in block diagram since the reference input is compared with a combination of all the states multiplied by the gain K and not with the output alone.

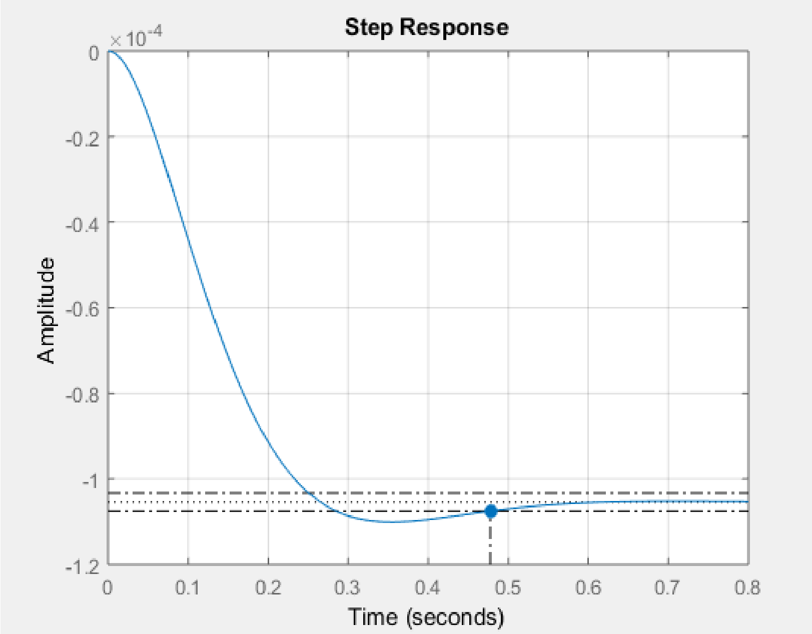

Fig 6.Step response of the system without using Pre-compensator gain G_N

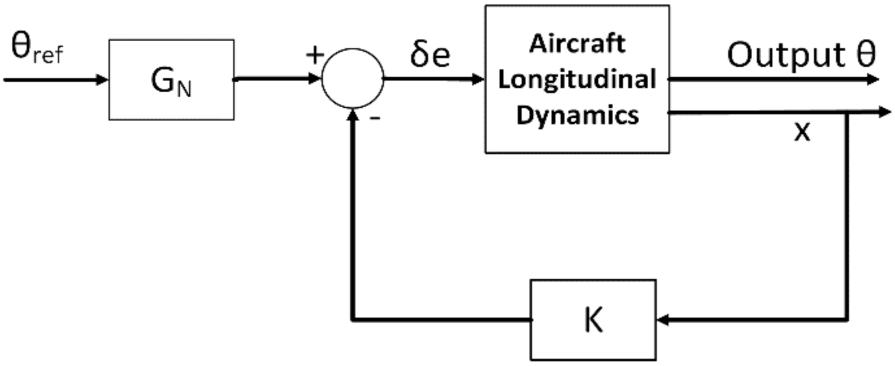

Fig 7. Block Diagram of the pitch control of aircraft using pre-compensator gain G_N

VIII. POLE PLACEMENT DESIGN RESULTS

In order to satisfy the desired transient response specifications of peak overshoot within 10 % and settling time within 4 seconds, the closed loop pole locations are chosen as $-2 \pm 0.4j$ and -6.

Using Pole Placement design, Feedback Gain K obtained is as $[-33.9859\ 17.53339 - 19.5807]$ and Pre-Compensator G_N is given by -33.9859 .The step response of the system using Pole Placement is shown in figure 8.

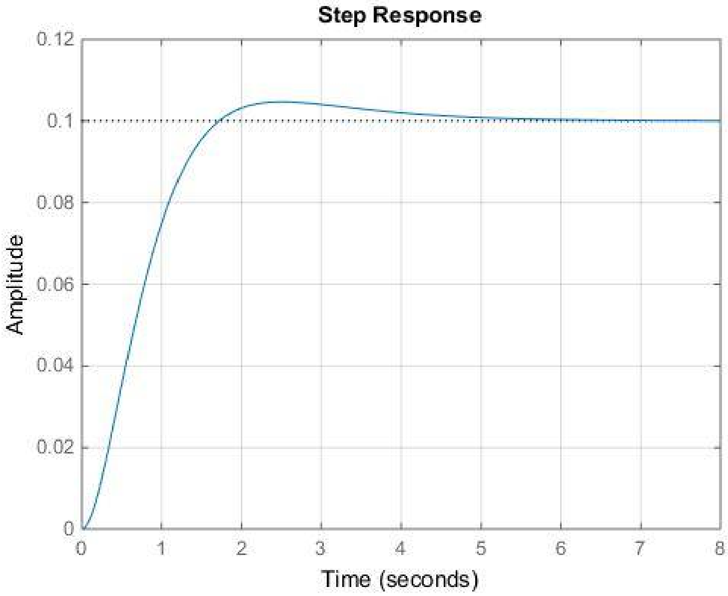

Fig 8. Step Response using Pole Placement

Using Pole Placement technique the performance characteristics obtained are as shown in Table 1.

Table 1. Performance characteristics of Pole Placement

Performance Specifications	**Value**
Rise Time	**1.1 s**
Peak Response	**0.105**
Settling Time	**3.98 s**
Overshoot	**4.66%**

IX. LQR DESIGN RESULTS

By fine tuning Q and R matrices as given in (16) using , the LQR controller gain K is obtained as shown in (17) with output along with 3 states pitch angle, angle of attack, pitch rate respectively.

$$Q=\begin{bmatrix} 90 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \ \& \ R=0.0001 \qquad (16)$$

$$K = [-948.6833 \quad 13.5169 \quad -100.6071] \qquad (17)$$

The Pre-Compensator Gain $G_N = -948.6833$ (18)

Using the control input δ_e given by (19) output pitch angle tracks the reference pitch command of 0.1 radian while the other two states are driven zero. The closed loop transfer function designed using LQR given by (20).The step response obtained using LQR design technique is shown in figure10. The Simulink Diagram of the system is as shown in figure 9.

Performance characteristics of LQR is obtained using Table.2

$$\delta_e = \theta_{ref} G_N - K \begin{bmatrix} \theta \\ \alpha \\ q \end{bmatrix} \qquad (19)$$

$$\delta_e = \theta_{ref} G_N - [k_1 \quad k_2 \quad k_3] \begin{bmatrix} \theta \\ \alpha \\ q \end{bmatrix} \qquad (20)$$

The closed loop transfer functions yields, $\frac{\theta}{\theta_{ref}} = \frac{155.74s+116.1197}{s^3+0.7455s^2+13.09098s+116.1197}$ (21)

The transfer function indicates that all the poles are on the left half of s plane($s= -0.7455, -8.78 \pm 8.87j$) and hence the closed loop system is stable.

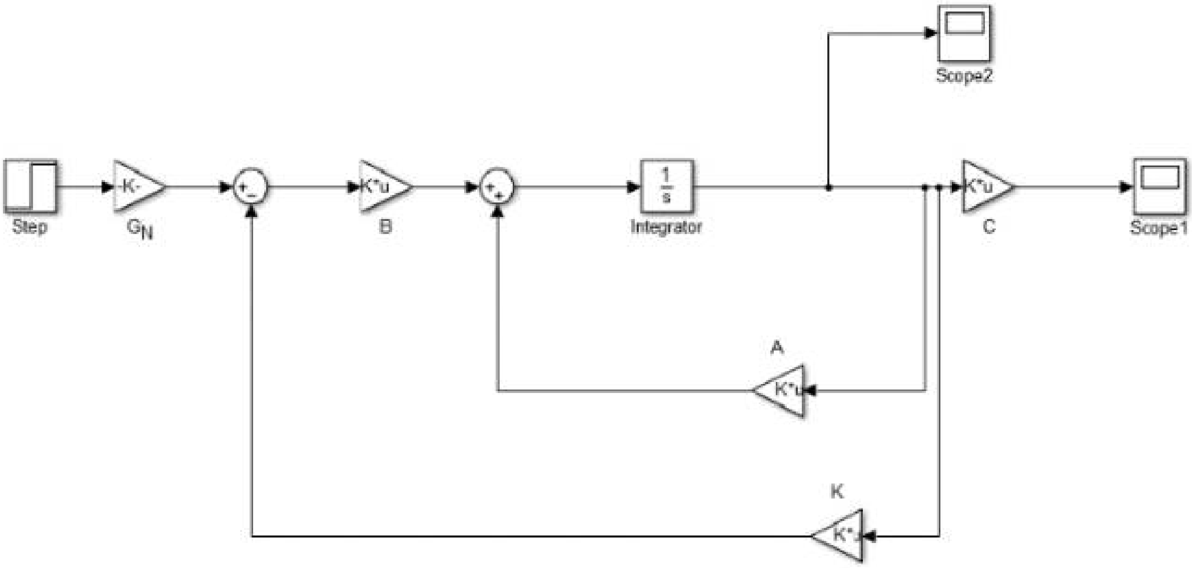

Fig 9. Simulink diagram of LQR

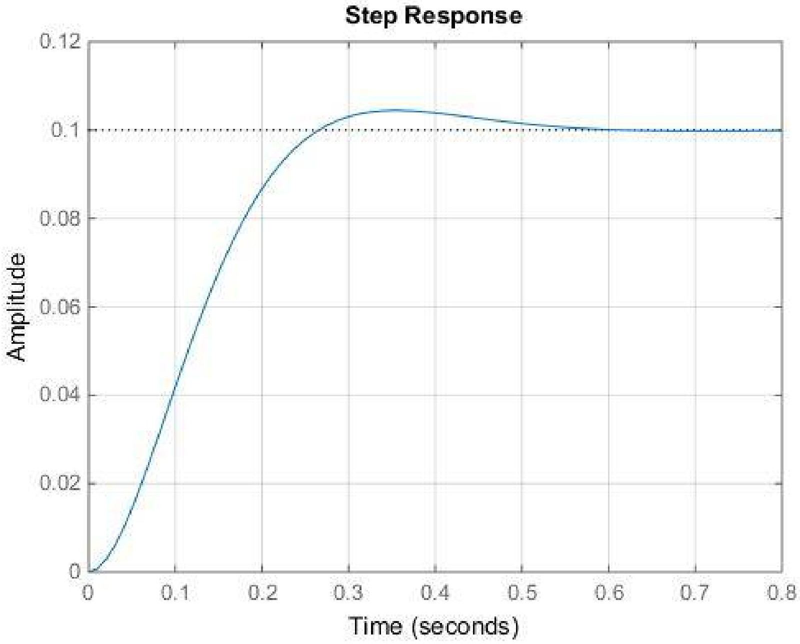

Fig 10. Step Response of LQR method with 0.1 radian step input

Table 2. Performance characteristics of LQR

Performance Specifications	Value
Rise Time	**0.172 s**
Peak Response	**0.104**
Settling Time	**0.478 s**
Overshoot	**4.44%**

Comparison of Performance Characteristics of Pole Placement and LQR is as shown in Table 3.

Table 3. Comparison between the performance characteristics of Pole Placement and LQR

Performance Specifications	Pole Placement	LQR
Rise Time	**1.1 s**	**0.172 s**
Peak Response	**2.5**	**0.104**
Settling Time	**3.98 s**	**0.478 s**
Overshoot	**4.66%**	**4.44%**

Hence it is evident that LQR design has better performance than conventional pole placement method.

X. CONCLUSION

LQR controller has been designed for the AFTI/F16 aircraft based on linearization of the state space model. The performance specifications achieved indicate that the system settles at a much faster rate using the proposed advanced linear control method rather than the conventional Pole Placement method. The overshoot obtained for a step input is found to be very minimal being a very critical requirement for the fighter aircraft. The output pitch angle tracks the desired reference input at steady state. The LQR technique attains the desired performance specifications achieving closed loop stability without increasing the order of the system. Comparative study has been carried out with the results obtained from Pole Placement technique. Simulation results validate that using LQR technique, an improved transient response specification has been achieved, the step response of the system with LQR settles at a much faster rate giving less overshoot compared to Pole Placement technique.

REFERENCES

[1] G. Joel Jacob and Kirubakaran P S B “Designing and Analysis Longitudinal Autopilot for a Jet Transport Aircraft’s Takeoff” *International journal of Engineering research &Technology (IJERT).*

[2] Dr S C Sharma and Dr Raminder Kaur “Longitudinal Autopilot for Different Aircrafts” International journal of advanced research in electrical electronics and instrumentation (an iso 3297:2007 certified organization)

[3] Jisha Shaji and Aswin R B “Pitch Control of Aircraft Using LQR & LQG Control” International journal of advanced research in electrical electronics and instrumentation (an iso 3297:2007 certified organization)

[4] Norman Nise “Control System Engineering” 7th edition.

[5] K.Ogata“ Modern Control Engineering” 5th edition.

[6] A.Nagoor Kani “Advanced Control Theory” 3rd edition.

[7] “Aircraft Pitch: State Space methods for controller design “Available: http://ctms.engin.umich.edu/CTMS/index.php?example=AircraftPitch&section=ControlStateSpace

[8] “Pole Placement Control Design “Available: https://nptel.ac.in/courses/101108047/module9/Lecture%2021.pdf

[9] “Introduction to Linear Quadratic Regulator”Available: https://www.youtube.com/watch?v=wEevt2a4SKI

[10] Sivaram Wijenddra, Manuel Vergara and Gunneet Singh, “Autopilot Design for an F16 Fighter Aircraft,” Available: https://users.encs.concordia.ca/~ymzhang/courses/MECH480-6091/CourseProjects&Labs/MECH6091/F2011/MECH6091_Presentation_Group2.pdf.