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Design and Implementation of Linear Algebraic Controller for Broom Balancing

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ABSTRACT: This paper discusses how the broom isdynamically stabilizable. It is suggested because; the dynamics of the Broom is analogous to the dynamics of Pitch and Yaw motion of a rocket, and robot arm motion. The broom is mounted on a moving cart. The nonlinear dynamics of the system is modeled and linearized. For the linearized unstable system, Linear Algebraic Controller is designed to balance the broom, and the simulation result shows that the broom is well balanced even when it is initially disturbed to $\pm 20^0$ angles.

KEYWORDS: Broom Balancing, Unstable Non-linearity, Cart and Broom motions, Broom Angular Position.

I.INTRODUCTION

Children try to balance a long slim wood and broom (which is used to clean floor) on their palm or index finger. With the broom exactly centered above motionless hand, and nothing pushes it to one side; it was balanced. At equilibrium, it can stay that way indefinitely, but in practice it never does. The slightest shift of the broom's center of gravity to one side causes unbalance. Any disturbance that shifts the broom away from equilibrium gives rise to forces that push the broom still farther from equilibrium, so it becomes unstable. Any object that has no base of supporthas an unstable equilibrium and tips over when disturbed. Instability stems from the fact that its center of gravity always descends when it is tipped and it releases gravitational potential energy as a result. The broom is unstable in motionless hand. But if the hand is moved, the broom can be stabilized. This will be done by endlessly moving the hand under the broom's center of gravity. If the broom starts to tip to the left, its handle could be moved to the left to place the handle under the broom's shifted center of gravity, i.e. it needs to constantly adjust the position of the hand to keep the broom upright. In that manner, the broom is kept returning to its equilibrium. Even though the equilibrium is naturally unstable, it can be kept up by helping it out and make it dynamically stable.
The Broom Balancing (installedon a cart) does basically the same thing. But, to simplify the problem, it can be required to move in one direction.It is a well-known example of nonlinear unstable control problem. The naturally unstable equilibrium corresponds to a state in which the broom points strictly upwards and, thus, requires a control force to maintain this position. The basic control objective of the broom problem is to maintain the forced stable equilibrium position when the broom initially starts atsome angle.

II. LITERATURE SURVEY

Broom balancing problem is used by many researchers with different names, pole balancing, pole – cart problem, inverted pendulum, stick balancer [12]. Different researchers used different control system approaches to sort out the problem [13]. This problem is standardized these days with technical advancement and publications [15]. Monorail system is one good application of this broom balancing system [14]. A free-swinging broom is inverted so the hinge is at its bottom and is attached to a cart, which moves back and forth. The challenge is to move the bottom of the broom back and forth to keep it from falling over. The cart is driven by an electric motor[1]. A controller controls the voltage applied to the electric motor according to current angular position of the broom. Someapplicationareas of Broom Balancing are Simulation for Dynamics control of Robotic Arm and Rocket,controlling a crane at construction site,

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missile guidance for military purpose, autopilot control (pitch, roll and yaw) of any air planfor stability (level flight) [16].Model of a Human Standing Still andinMusic instrument- Metronometo correct music rhythm.

In this paper, system setup for broom balancing generally includes the mechanical and electrical parts. The primary mechanical considerations are the cart and its propulsion, and the behavior of the broom. The electrical part of system setup includes sensor, signal amplifier, controller and motor drive circuits as shown in Fig.1.

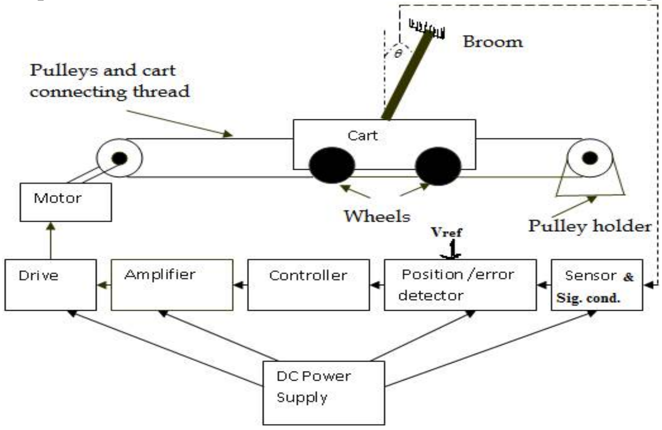

Fig.1. Setup for Broom Balancing

III.METHOD

When the broom is at unstable position, it has two motions. First, the broom falls to its naturally stable equilibrium (down) position, i.e. leading motion- motion of center of gravity (cg) of broom with respect to a cart. The second is motion of the cart and the broom with respect to around. The broom moves with the cart when the cart moves to keep the broom at upright position due to leading motion.

Dynamic Model of the Broom**:**

Acronyms

G	= *gravitational acceleration*
gc	= *broom gravity center*
PP	= *pivot point*
X_{gc}	= *X coordinate of broom gravity center*
Y_{gc}	= *Y coordinates of broom gravity center*
X_p	= *X coordinate of pivot point*
Y_P	= *Y: coordinate of pivot point*
F_H	= *horizontal direction Force*
F_V	= *vertical direction Force*
l	= *½ length of broom*
m	= *broom mass*
M	= *Total mass of the system*
θ	= *Angle of broom measured from vertical.*
B_r	= *Viscous damping constant at pivot point of broom*
I	= $ml^2/3$*: Moment of inertia for uniform rod of broom*
V_a	= *motor armature voltage*
E	= *motor armature Emf*
R_a	= *motor armature resistance*

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T_d = *motor developed torque*
F = *force applied to cart*
φ = *motor shaft angle*
r = *pulley's effective radius*
K_m = *voltage and torque constant of motor*
B_m = *motor viscous damping*
ω_m = *motor angular speed* ($\omega_m = \dot{X}_{pp}/r$)
B_c = *cart viscous damping*
J_m = *motor inertia*
M_c = *cart's mass (M-m)*
J = *combined inertia of motor and cart* $(J = J_m + Mr^2)$
B = *combined viscous damping of motor and cart*
= $(B_c r^2 R_a + B_m R_a + K_m^{\ 2})/JR_a$
K_c = *viscous damping constant* = $k_m r / R_a J$

For the whole system mathematical model determination, system seen in Fig.1 is divided in to two as shown in Fig.2 and Fig.3.Components of forces applied on the broom are shown in figure Fig.2.

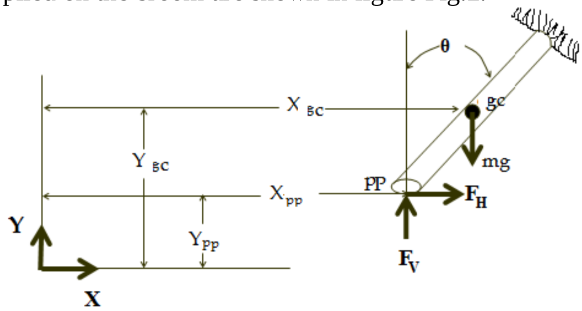

Fig2.Diagram shows components of forces exerted on broom

Coordinates of point cg of the broom in terms of half of its length l, point pp, and angle θ are

$$Y_{gc} = Y_{pp} + l\cos\theta \qquad (2.1)$$

$$X_{gc} = X_{pp} + l\sin\theta \qquad (2.2)$$

Equation of Components of Forces applied on the Broom: When the downward-directed gravitation force and the opposing upward-directed force provided by pivot are not aligned, their resultant causes the application of a torque that tilts the broom. The magnitude of the torque increases as the angle of the falling broom increases relative to vertical axis.

1. Sum of forces in X direction:

$$\sum F_X = m\ddot{X}_{gc} \qquad (2.3)$$

$$F_H = m\,\ddot{X}_{PP} + ml\cos\theta\,\ddot{\theta} - ml\sin\theta\dot{\theta}^2 \qquad (2.4)$$

2. Sum of forces in Y direction:

$$\sum F_Y = m\ddot{Y}_{gc} \qquad (2.5)$$

$$F_V = ml\sin\theta\,\ddot{\theta} - ml\cos\theta\dot{\theta}^2 + mg \qquad (2.6)$$

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3. Sum of moments about gravity center:

$$\sum M_{gc} = I\ddot{\theta} + B_r\dot{\theta} \quad (2.7)$$

$$F_v l \sin\theta - F_H\, l \cos\theta = I\ddot{\theta} + B_r\dot{\theta} \quad (2.8)$$

Substitute equations (2.4) and (2.6) into (2.8), we have

$$(I + mI^2)\ddot{\theta} + B_r\dot{\theta} - mgl \sin\theta = -ml\,\ddot{X}_{PP} \cos\theta \quad (2.9)$$

This is equation for the motion of abroom pivoted on the cart, which relatesthe cart and thebroom positions.

The Cart and its Actuator (Motor) Model: Figure 3 below represents the way the cart driven by the motor. In modelingmotion of the Cart and Motor, we need

Basic Equations of Motor

$$V_a = E + i_a R_a \quad (210)$$

$$E = K_m\dot{\varphi} \quad (2.11)$$

$$i_a = \frac{1}{K_m} T_d \quad (2.12)$$

Combine equations (2.10) and (2.12)

$$V_a = K_m\dot{\varphi} + \frac{1}{K_m} T_d R_a \quad (2.13)$$

TheTorque developed by motor is

$$T_d = Jw_m + B_m w_m + Fr \quad (2.14)$$

Where

$$F = M_c\ddot{X}_{PP} + B_c\dot{X}_{PP} + F_H \quad (2.4)$$

And F_H is given by

$$= M_c\ddot{X}_{PP} + B_c\dot{X}_{PP} + m(\ddot{X}_{PP} + l\cos\theta\ddot{\theta} - l\sin\theta\dot{\theta}^2$$

$$= M_c\ddot{X}_{PP} + B_c\dot{X}_{PP} + m\,l\,(\cos\theta\ddot{\theta} - l\sin\theta\dot{\theta}^2) \quad (2.15)$$

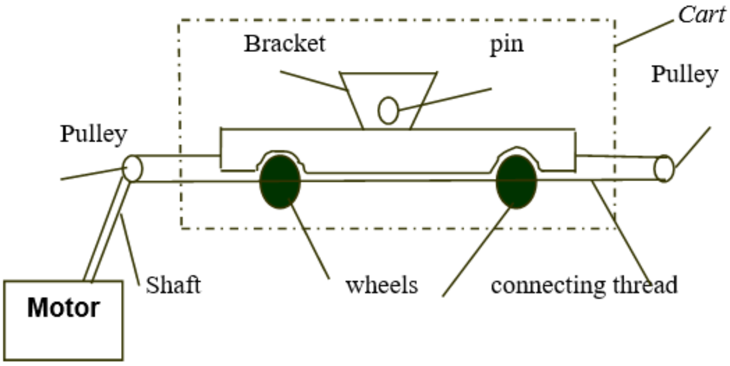

Figure 3. The cart and its drive system

Since $M = M_c + m$, and Substituting (2.15) into (2.14),

$$T_d = J_m W_m + B_m w_m + (M\ddot{X}_P + B_c\dot{X}_P + m\,l(\cos\theta\ddot{\theta} - l\sin\theta\dot{\theta}^2))r \quad (2.16)$$

Since $\ddot{\varphi} = \dot{w}_m = \frac{\ddot{X}_P}{r}$ is motor shaft acceleration (2.17)

Substituting (2.16) and (2.17) into (2.13), after simplification, the motor armature input voltage is

$$V_a = \frac{1}{K_c}[\ddot{X}_{PP} + B\dot{X}_{PP}\frac{m\,l\,r^2}{J}(\cos\theta\ddot{\theta} - l\sin\theta\dot{\theta}^2)] \quad (2.18)$$

Equations (2.9) and (2.18) represent the non-linear model of the broom balancing system. The non-linearity is because of the trigonometric part and the quadratic term$\dot{\theta}^2$. Since the goal in this paper is to keep the broom upright at $\theta = 0^0$ from vertical axis (i.e. vertically upward), linearization of the system is considered about the upright equilibrium point. Assume that $\theta = \pi - \emptyset$; $\emptyset$ represents small angle from vertical position.Let $\sin\theta \cong \emptyset, cos\theta \cong -1$, and assume

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that $\dot{\theta}$will kept small so that its square is almost zero. Using this approximation the linearized dynamic model of the system is given by equations (2.19) and (2.21)

$$V_a = \frac{1}{K_c}[\ddot{X}_{PP} + B\dot{X}_{PP} - \frac{m\,l\,r^2}{J}\ddot{\emptyset}] \quad - \qquad (2.19)$$

This equation relates the electrical variable-motor armature input voltage directly to thenon-electrical variables-cart and the broom positions.

For $I = \frac{m\,l^2}{3}$simplification of equation (2.9) is

$$\ddot{\emptyset} + \frac{3B_r}{4ml^2}\dot{\emptyset} - \frac{3g}{4l}\emptyset = \frac{3}{4l}\ddot{X}_{PP}$$

(2.20)

Let $2\xi w_m = \frac{3B_r}{4ml^2}, w_m^2 = \frac{3g}{4l}, K_{pl} = \frac{3}{4l}\ ; then$

$$\ddot{\emptyset} + 2\,\xi w_n\dot{\emptyset} - w_n^2\emptyset = K_{pl}\ddot{X}_{PP} \qquad (2.21)$$

2.3 Determination of Transfer Functions ofthe System

From equation (2.19), solving for $\ddot{X}_{pp}$ we have:

$$\ddot{X}_{PP} = K_cV_a - B\dot{X}_{PP} + \frac{m\,l\,r^2}{J}\ddot{\emptyset} \qquad (2.22)$$

Substitute (2.22) into (2.21) and solve for$\ddot{\emptyset}$:

$$\ddot{\emptyset} = \frac{-JK_{pl}B}{J-K_{pl}mlr^2}\dot{X}_{pp} + \frac{Jw_n^2}{J-K_{pl}mlr^2}\emptyset - \frac{2J\xi w_n}{J-K_{pl}mlr^2}\dot{\emptyset} + \frac{K_{pl}K_cJ}{J-K_{pl}mlr^2}V_a \qquad (2.23)$$

If we substitute (2.23) into (2.22), $\ddot{X}_{pp}$ will be obtained as:

$$\ddot{X}_P = \frac{-BJ}{J-K_{pl}mlr^2}\dot{X}_{pp} + \frac{mlr^2w_n^2}{J-K_{pl}mlr^2}\emptyset - \frac{2\xi W_nmlr^2}{J-K_{pl}mlr^2}\dot{\emptyset} + \frac{K_cJ}{J-K_{pl}mlr^2}V_a \qquad (2.24)$$

Taking Laplace transform of equation (2.21), the transferfunction between $X_{pp}(s)$ and $\phi(s)$ can be obtained as

$$G_{\emptyset X}(S) = \frac{\emptyset(S)}{X_{PP}(S)} = \frac{K_{pl}S^2}{S^2+2\xi w_nS-w_n^2} \qquad (2.25)$$

Also the Laplace transform of equation (2.24) is

$$X_{PP}(S) = \frac{-(2\xi W_nmlr^2S-mlr^2W_n^2)\emptyset(S)+K_CJV_a(S)}{(J-K_{pl}mlr^2)S^2+BJS} \qquad (2.26)$$

Multiplying equation (2.25) and (2.26), and solving for $\frac{\phi(s)}{V_a(s)}$; after simplification we have:

$$G_{\emptyset}(S) = \frac{\emptyset(S)}{V_a(S)} = \frac{K_{pi}K_cS}{\left(1-\frac{K_{pl}mlr^2}{J}\right)S^3+(B+2\xi w_n)S^2+(2B\xi w_n^2)S-Bw_n^2} \qquad (2.27)$$

The block diagram for the closed-loop transfer function of the system without controller is shown in figure 4. This is done by feeding back the angular position of the broom using sensor (sensescurrent angle and coverts it to voltage which is compared with reference voltage).

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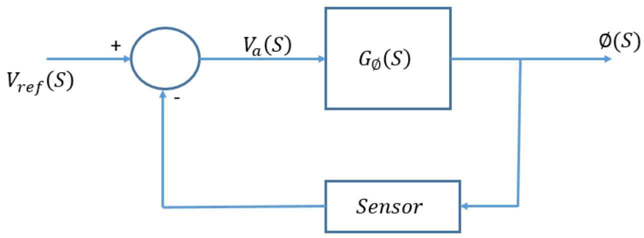

Fig4. Block diagram for closed loop transfer function of the system

Determination of the Parameters for the Broom Balancing System: To determine the stability of the broom, the parameters of the overall system are required. The parameters of the system such as mass of the cart- M, mass of the broom- m, length of the broom- L, radius of the pulley- r and motor armature resistance- R_a are all measured directly. The motor constant parameter- K_m was determined experimentally. The values of required parameters are listed in table1.

Table 1: Parameters for the broom balancing System

Parameter	Description	Values
M	Total mass of the system	1kg
M	Mass of rod	0.1kg
L	Length of rod	0.5m
ℓ	Length of rod from bottom to its centroid	0.25m
r	Pulley radius	15mm
B_c	Viscous damping of the cart	0.1N sec/m
B_r	Viscous damping at pivot point of broom	0.05Nm sec/rad
G	Gravitation acceleration	9.8m/sec^2
K_m	Motor constant for both torque and voltage	0.01Nm/A
J_m	Motor inertia	0.01Nm sec^2/rad
B_m	Motor viscous damping	0.1Nm sec/rad
R_a	Motor armature resistor	1Ω

Inserting the parameter values in the table 1 into equation (2.27) the open-loop transfer functionrelating the broom angular position and required motor armature voltageis obtained as follows.

$$G_\emptyset(S) = \frac{\emptyset(S)}{V_a(S)} = \frac{0.0579S}{S^3+15.67\,S^2-60\,s-1136} \qquad (2.28)$$

As it will be seen in the next section, the dynamic equation ofbroom is unstable.Using Matlab, the pole-zero map and step response of the linearized model of broom (in open loop configuration) are shown in figure 5& 6 respectively, which indicatethe broom is unstable/ unbalanced.The step response of the uncontrolled closed-loop with unity feedback is shown in the Fig7. The response is almost identical with the open-loop step response. This indicates the system cannot be stabilized by simple unity feedback.

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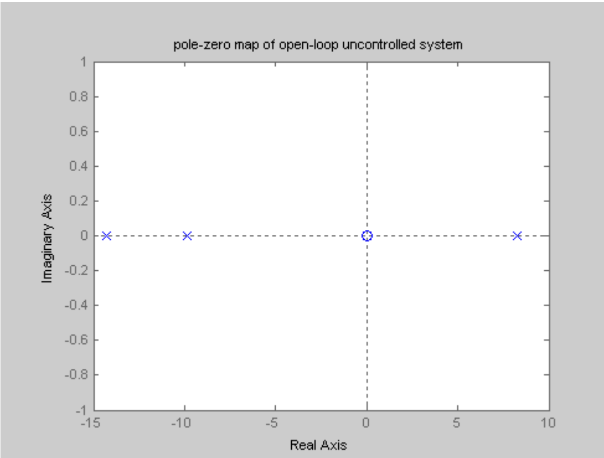

Fig5. Poles and zeroes map of open loop transfer function G_{Φ}

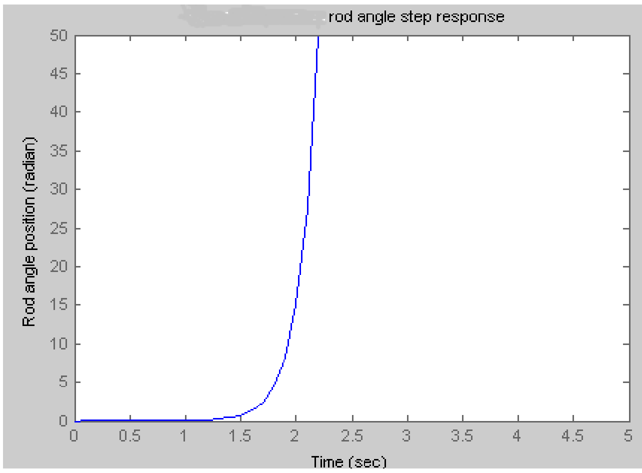

Fig6. Step response of the open-loop transfer function $G_{\Phi}(s)$.

Reshaping of the system the response is necessary by incorporating proper controller in to the system to shift the unstable pole into the left half plane (stable region) of the s-plane.

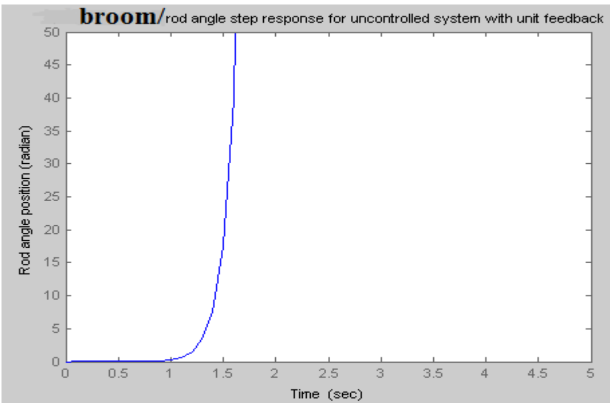

Fig7. Step response for closed-loop uncontrolled broom balancing system with unity feedback.

IV. CONTROLLER DESIGN

First a desired closed loop transfer function- $\mathbf{G_0(s)}$ is determined, and then solved for required controller. That is why it is called a linear algebraic method for controller design. It tries to satisfy a total prescribed system transfer function for single-input - single-output (SISO) systems.

For a plant with a proper transfer function $G_{\emptyset}(S) = \frac{N(S)}{D(S)}$, with N(s) and D(s) coprime; and the transfer function $G_{\emptyset}(S) = \frac{N_0(S)}{D_0(S)} = \frac{\emptyset(S)}{V_{ref}(S)}$ meets the design specifications, where $V_{ref}(s)$ and $\phi(s)$ are reference input and output of closed-loop system, respectively. The desired closed-loop transfer function $G_0(s)$ is implementable if:

a) Degree of $D_o(s)$ minus degree of $N_o(s) \geq$ degree of D(s) minus degree of N(s).
b) All closed loop right hand plan zeros of N(s) are retained in $N_o(s)$.
c) $D_o(s)$ is Hurwitz, i.e. all its roots are in the left hand plan.

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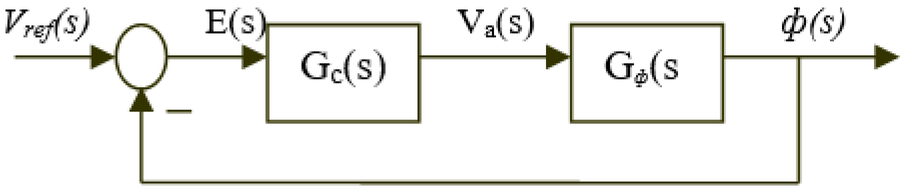

Fig. 8Block diagram of unity feedback control system

The simpler attemptforapplication of the Linear Algebraic Method to determine controller transfer function $G_c(s)$ with a unity feedback system, the overall desired closed-loop transfer function in Fig. 8 is representedas

$$G_0(S) = \frac{\emptyset(S)}{V_{ref}(S)} = \frac{N_0(S)}{D_0(S)} = \frac{G_c(S)G_\emptyset(S)}{1+G_c(S)G_\emptyset(S)} = \frac{N_c(S)N(S)}{D_c(S)D(S)+N_c(S)(S)N(S)} \qquad (3.1)$$

For the unity feedback arrangement, for $G_\emptyset(S), G_c(S) and\ G_0(S)$ are as defined above and with degree of $N(S)$ ≤degree of $D(s)$ = n, from equation (3.1), we have

$$D_c(S)D(S) + N_c(S)N(S) = D_0(S) \qquad (3.2)$$

This is a polynomial equation where$D_c(S) and\ N_c(S)$are unknown polynomials. Equation (3.2) can be solved using the Sylvester matrix technique [1, 6], expanding the polynomials as;

$$\begin{aligned} N(S) &= N_0 + N_1S + \ldots + N_nS^n \\ D(S) &= D_0 + D_1S + \ldots + D_nS^n;\ D_n \neq 0 \\ N_c(S) &= N_{c0} + N_{c1}S + \ldots + N_{cm}S^m \\ D_c(S) &= D_{c0} + D_{c1}S + \ldots + D_{cm}S^m \end{aligned} \qquad (3.3)$$

Substituting (3.3) in (3.2), we have

$$\begin{aligned} D_0(S) = {} & (D_{c0} + D_{c1}S + \cdots + D_{cm}S^m) * (N_{c0} + N_{c1}S + \cdots + N_{cm}S^m) \\ & +(N_{c0} + N_{c1}S + \cdots + N_{cm}S^m * (N_0 + N_1S + \cdots + N_nS^n) \qquad (3.4a) \\ = {} & F_0 + F_1S + \ldots + F_{n+m}S^{n+m} \qquad (3.4b) \end{aligned}$$

The polynomial coefficients $F_0, F_1 \ldots F_{n+m}$are obtained from pole-placement of $G_0(S)$. Equating coefficients of like power of "S" in (3.4a) and (3.4b) leads to:

$$\begin{aligned} F_0 &= D_{c0}D_0 + N_{c0}N_0 \\ F_1 &= D_{c0}D_1 + D_{c1}D_0 + N_{c0}N_1 + N_{c1}N_0 \\ &\ \ .\qquad\qquad . \\ &\ \ .\qquad\qquad . \\ &\ \ .\qquad\qquad . \\ F_{n+m} &= D_{cm}D_n + N_{cm}N_n \end{aligned} \qquad (3.5)$$

This can be rewritten in the form of Sylvester matrix as follows.

$$\begin{bmatrix} D_0 & N_0 & 0 & 0 & & 0 & 0 \\ D_1 & N_1 & D_0 & N_0 & & \bullet & \bullet \\ \bullet & \bullet & D_1 & N_1 & & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & & 0 & 0 \\ D_n & N_n & \bullet & \bullet & \bullet\ \bullet\ \bullet & D_0 & N_0 \\ 0 & 0 & D_n & N_n & & D_1 & N_1 \\ \bullet & \bullet & 0 & 0 & & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & & \bullet & \bullet \\ 0 & 0 & 0 & 0 & & D_n & N_n \end{bmatrix} \begin{bmatrix} D_{c0} \\ N_{c0} \\ D_{c1} \\ N_{c1} \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ D_{cm} \\ N_{cm} \end{bmatrix} = \begin{bmatrix} F_0 \\ F_1 \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ F_{n+m} \end{bmatrix} \qquad (3.6)$$

Equation (3.6) has solution if the matrix of coefficients has full row rank. In other wards the following condition hold true.

$$n + m + 1 \leq 2(m + 1) or\ n - 1 \leq m \qquad (3.7)$$

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To achieve arbitrary pole-placement, the degree of controller $G_c(s)$ in the unity feedback configuration must be m = n-1 or higher. If it is less than n-1, it may be possible to assign some of the poles but not all. The degree of the characteristic equation $D_0(s)$ of transfer function $G_0(s)$ is $n + m$. The unknowns in (3.2) can be determined by (3.6) so that for specified performance and coefficients, $G_c(s)$ isexpressed as:

$$G_c(S) = \frac{N_c(S)}{D_c(S)} = \frac{N_{c0}+N_{c1}S+...+N_{cm}S^m}{D_{c0}+D_{c1}S+...+D_{cm}S^m} \tag{3.8}$$

For the stabilization of the system, the closed-loop poles of the transfer function $G_0(s)$ must be in the left hand side of s-plane. The dominant poles of characteristic equation of $G_0(s)$ in the complex root plane approximately determine transient performance and stability of a linear time invariant system. The damping ratio and the natural frequency resulting from the dominant poles of a more than two order system can be used to determine the boundary of a desired region in the complex plane within which all the roots of characteristic equation must be located. Assuming that dominant poles are complex roots and given as:

$$S_{1,2} = -\xi_0 w_0 \pm jw_d \tag{3.9}$$

Where$\xi_0$is damping ratio
$W_0$is un-damped natural frequency
$w_d = w_0\sqrt{(1-\xi_0^2)}$is damped frequency

We have to design a controller so that the step response of the closed-loop system meets the settling time (T_S),delay time (T_d), maximum overshoot (M_p), damping ratio and steady-state error specifications [3]. The settling time and delay timeare measures of the speed of the system response. Increasing the natural frequency-w_0 can increase the speed of the response. In general, the maximum overshoot decreases as the damping ratio increases. However, as damping ratio increases, the delay time increases which causes the response of the system sluggish. Therefore, suitable value of damping ratio should be selected to minimize the overshoot without affecting the speed of response of the system.

To calculate dominant poles, let $\xi_0 = 0.85$ and T_s= 0.80 Sec

$\xi_0 w_0 = \frac{4}{0.8} = 5$ $w_0 = 5.88\, rad/Sec$ $w_d = 3.10\, rad/Sec$ $M_p = 0.63\,\% <$ 10%

Using arbitrary pole-placement with unity feedback, the controller to stabilize the broom is designed next. From (2.28), the unstable pant transfer function is

$$G_\emptyset(S) = \frac{\emptyset(S)}{V_a(S)} = \frac{0.0579\,S}{S^3+15.67S^2+60S+1136} = \frac{N(S)}{D(S)} \tag{3.11a}$$

The numerator and denominator polynomials of $G_\phi(s)$ can be written as:

$$N(S) = 0\,S^3 + 0\,S^2 + 0.0579\,S + 0 \tag{3.11b}$$

$$D(S) = S^3 + 15.67\,S^2 - 60\,S - 1136 \tag{3.11c}$$

The order of the plant transfer function is n=3, and it needs controller $G_c(s)$ of the order m = n-1 = 2. This shows the degree of the characteristic equation $D_0(s)$ of the overall transfer function $G_0(s)$ is$n + m = 5$. Based on the performance specifications setting taken above, the dominant poles are

$$S_{1,2} = \xi_0 w_0 \pm jw_d = -5 \pm j3.1$$

And the others arbitrary stable poles are selected as$S_{3,4} = -16 \pm j\,9$ and $S_5 = -2$, then polynomial of the characteristic equation $D_0(s)$ is

$$\begin{aligned} D_0 &= (S + 5 + j\,3.1)(S + 5 - j3.1)(S + 16 + j\,9)(s + 16 - j\,9)(s + 2) \\ &= S^5 + 44S^4 + 785.75S^3 + 6420.83S^2 + 27122S + 341767 \\ &= F_0 + F_1S + \ldots + F_5S^5 \end{aligned} \tag{3.11d}$$

Putting equations (3.11b) to (3.11d) in the matrix form to determine the unknown coefficients of polynomials $N_c(s)$ and $D_c(s)$ so that the controller $G_c(s)$ and the overall transfer function $G_0(s)$ can be specified.

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$$\begin{bmatrix} -1136 & 0 & 0 & 0 & 0 & 0 \\ -60 & 0.058 & -1136 & 0 & 0 & 0 \\ 1567 & 0 & -60 & 0.058 & -1136 & 0 \\ 1 & 0 & 1567 & 0 & -60 & 0.058 \\ 0 & 0 & 1 & 0 & 1567 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} D_{c0} \\ N_{c0} \\ D_{c1} \\ N_{c1} \\ D_{c2} \\ N_{c2} \end{bmatrix} = \begin{bmatrix} 3417476 \\ 2712233 \\ 642083 \\ 78575 \\ 44 \\ 1 \end{bmatrix}$$

(3.12)

X Y F

The unknown coefficients are determined using matlab command Y = inv(X)*F.

i.e.

$$Y = \begin{bmatrix} D_{c0} \\ N_{c0} \\ D_{c1} \\ N_{c1} \\ D_{c2} \\ N_{c2} \end{bmatrix} = \begin{bmatrix} -30.08 \\ 993098.62 \\ 28.33 \\ 168013.53 \\ 1 \\ 7460 \end{bmatrix} \quad (3.13)$$

Therefore the desired closed loop and controller transfer functions are:

$$G_0(S) = \frac{432S^3+97279S^2+5750041S}{S^5+44S^4+785.75S^3+6420.83S^2+27122S+341767} \quad (3.14)$$

$$G_0(S) = \frac{993098.6+168013.5\,S+7460S^2}{-30.8+28.33S+S^2} = \frac{E(S)}{U(S)} \quad (3.15)$$

Where *U(s)* and *E(s)* are control and error signals as indicated in Fig.8 and (3.15) can be rewritten as:

$$[-30.08s^{-2} +28.33s^{-1}+1]U(s)$$
$$= [993098.63s^{-2} +168013.53s^{-1}+7460]\,E(s) \quad (3.16)$$

If like power of s are collected together we get

$U(s) = s^{-2}[30.08\,U(s) + 993098.63\,E(s)] + s^{-1}[-28.33\,U(s) +168013.53\,E(s)] + 7460\,E(s)$(3.17)this can be realized as Fig.9. Its synthesized practical circuit is a part of total circuit, which is used to stabilize the broom.

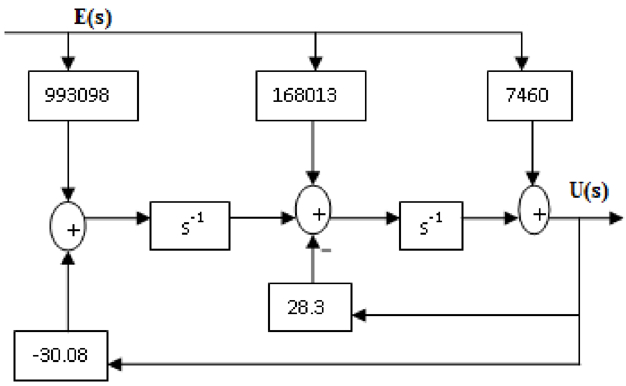

Fig.9Direct form realization of controller $G_c(s)$

V.RESULT AND DISCUSSION

The simulation result for step response of $G_0(s)$ is given by Fig.10. From the plot we get the following result.

The overall system is stabilized.

The control design meets performance specifications

- Maximum (peak) value =1.24 and settling time-T_s= 2.21sec due to arbitrary pole placement.
- Zero final value

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Using algebraic technique discussed above, the stability of the broom can be satisfactorily achieved by selection of arbitrary stable poles of overall transfer function of the system. And the performance specifications (large peak value and large settling time) of the overall system can be improved by proper selection of the poles.

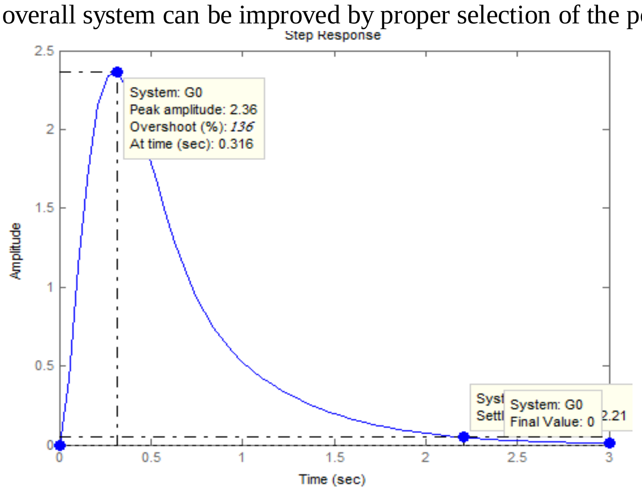

Fig. 10Stabilized broom angle step response.

From equation (3.14), the current angular position of the broom is expressed in s-domain in terms of its initial value- $\phi(0)$ (from vertical axis)and reference input voltage- $V_{ref}(s)$ and it is used to simulate effect of different values of $\phi(0)$ on the stabilized system as shown in Fig.11.

$$\emptyset(S) = \frac{(S^4+44S^3+7858S^2+6421S+27120)\emptyset(0)+(432S^3+97279S^2+5750041S)V_{ref}(S)}{S^5+44S^4+785.75S^3+6420.83S^2+27122S+341767} \quad (4.1)$$

The simulation result of step response of angular position of the broom for different values of initial angle $\phi(0) = 10$, 20 and -20 (in degree) introduced between vertical axis and broom is shown below. From this plot, it is observed that the error or initial displacement angle of broom goes to some small final value (1.23^0) which indicates the broom returns to its balance (upright) position.

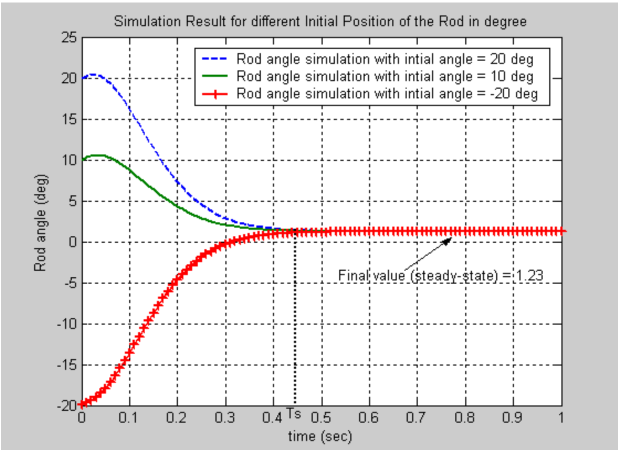

Fig.11Step response of broom angular position for different its initial angle

The frequency response simulation result for the combined transfer function of the plant and controller ($G_\Phi(s)\,G_c(s)$)is shown by bode diagram in the figure 12. The response shows closed loop system is well stable.

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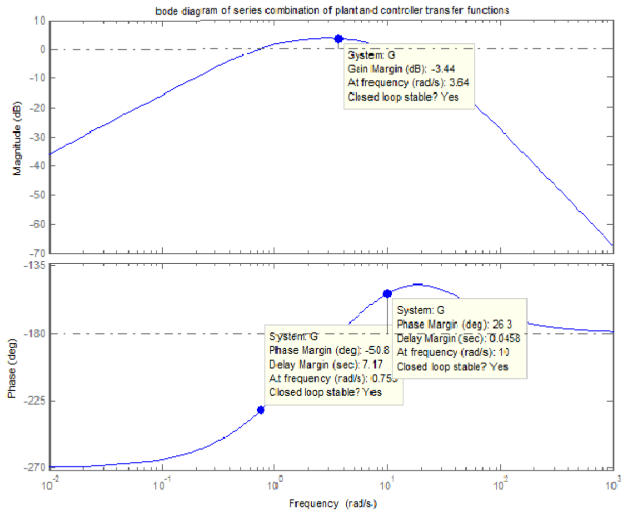

Fig.12 Frequency response simulation result

Practically the controller is feasible. The controller given in Fig.9, sensor and motor drive parts are practically realized in Fig.13. The controller is ringed from resistances and operational amplifiers. During practical test, the result shows the broom is satisfactorily stabilized; and somewhat amplifier saturation is seen.

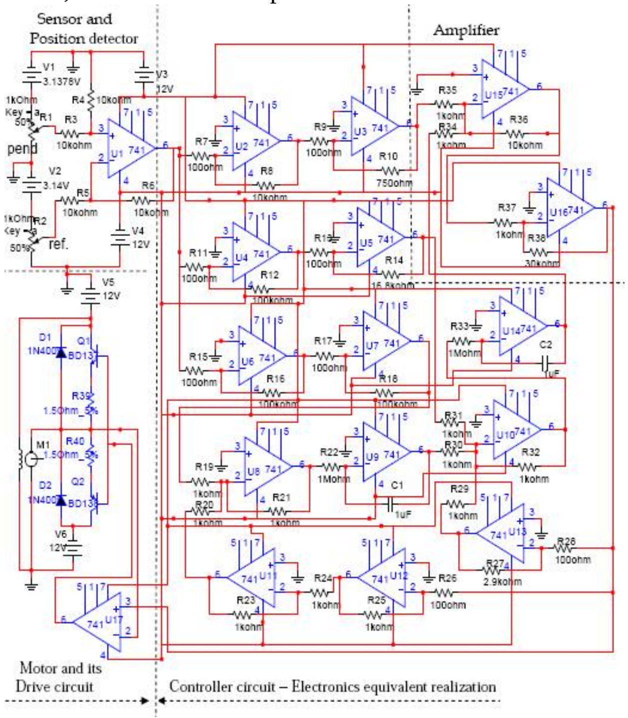

Fig.13 Complete circuit (controller, sensor and motor drive) realization

VI.CONCLUSION

The objective of this paper is to design controller that keeps the broom upright at its naturally unstable equilibrium position. This work, system modeling, linearization, analysis of both uncontrolled and controlled systems, and simulation of the overall system has been done successful. The analysis of system indicates that the system is unstable, that's why proper stabilizer is designed. The controller design methods are based on the transfer function approaches. The controllersatisfies both the stability of the system and design performance specification settings. Also the simulation result shows that for $\pm 20^0$-initial angle of the broom, the system is stabilized by the controllerwith in o.5 second and steady state value of 0.02rad (1.23 deg) for the broom is initially released from near $\pm 20^0$. This is due to for small deviation in broom angle; the sensor output is almost not changed. For the broom is at upright position, the output of controller is zero. In this case the controller does not get biasing signal. Broom Balancing provides a chance of designing a controller for a system that has good dynamic behavior. Hence the consideration for the transient response is emphasized. Another important point is that the broom on the cart provides a means for learning about electromechanical system.

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