Analysis of Bifurcation in Quadratic Buck Converter

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ABSTRACT: This paper presents the analysis of a bifurcation in quadratic buck converter. This class of converter offers very wide range of DC voltage conversion ratio enabling higher switching frequencies. In this paper, analysis of the quadratic buck converter in discontinuous conduction mode (DCM) is presented and corresponding simulation results obtained using MATLAB software. Pulse train control quadratic buck converter operated in DCM is analysed for bifurcation. Non linear dynamics of DCM quadratic buck converter under different input voltage and load conditions are examined and the simulation results are given.

KEYWORDS: Quadratic buck converter, Pulse train control, Bifurcation, Non linear dynamics.

I. INTRODUCTION

DC-DC converters continue to evolve as an indispensable part of source in many electronic circuits. The switching frequency of DC-DC converters can be suitably selected to offer the desired output voltage, and the switches allowed to turn ON and OFF in tune with the design specifications. However in real time applications, the supply voltages and the loads usually vary, necessitating a suitable corrective control action to stabilize the system in the desired operating state.

A very low voltage output from a conventional DC/DC buck converter demands for a very small switching duty ratio. But there is a limit on the minimum on time of an active switch and this may lead to an implementation problem. In cascade configuration, the conversion ratio is increased by the order of number of stages of converters. However, one of the main disadvantages of cascaded converters is that the overall efficiency is reduced by losses in switching devices. In the case of the quadratic buck converter, the voltage conversion ratio has a quadratic dependence on the duty ratio and is electrically equivalent to two buck converters connected in cascade with only one active switch. The behaviours of the converter operating in discontinuous conduction mode (DCM) have been reported here.

Pulse train (PT) control technique, which realizes the output voltage regulation of switching dc–dc converters by applying high-power control pulses or low-power control pulses with discrete duty ratios. In VMPT controller, the duty ratios of high-power control pulse and low-power control pulse are preset. This paper exhibits idea of voltage mode pulse train control applied to the quadratic buck converter for the bifurcation analysis.

Section I deals with the introduction to non linear dynamics and the basics of quadratic buck converter. The objective of the project is also highlighted in this section. Section II presents the analysis of quadratic buck converter with its various modes of operation in discontinuous conduction mode (DCM). The state space equations for the respective modes are also derived. Section III explains the voltage mode pulse train control applied to the quadratic buck converter for the voltage regulation. The control strategy for generation of the pulses corresponding to the required output power is also presented. Section IV shows the simulated results of the quadratic buck converter in discontinuous conduction mode. It also includes the waveform for different resistance load with the VMPT controller. Section V summarizes the conclusion.
II. OPERATION OF QUADRATIC BUCK CONVERTER IN DCM

During the last few years, a great number of applications for quadratic buck converters (QBCs) have been reported. In a QBC with a single switch, the dc conversion ratio has a quadratic dependence of the duty ratio. Its dynamical behaviour is similar to two cascaded buck converters but using one active switching device.

A. Modes of Operation in Discontinuous Conduction Mode:

The circuit diagram of a single-switch quadratic buck converter is shown in Fig. 1. The converter consists of two LC filters, one active switch (Q) and three passive switches (D₁, D₂, D). If the converter is operated at fixed switching frequency, the state-space averaged model of the quadratic buck converter can be described as \(d(t)\) is the duty ratio, \(V₁\) is the supply voltage, \(L₁\) and \(L₂\) are the inductances of the first and second stages of the converter, \(C₁\) and \(C₂\) are the capacitances of the first and second stages of the converter, \(R\) is the loading resistance, \(i_{L₁}\) and \(i_{L₂}\) are the currents of inductors \(L₁\) and \(L₂\), and \(V_{C₁}\) and \(V_{C₂}\) are the voltages across the capacitors \(C₁\) and \(C₂\) respectively.

When the QBC is operated in discontinuous conduction mode (DCM) it exhibits three modes. The first two modes are similar to the continuous conduction mode. In mode III only the output capacitor feeds the load.

B. Mode I:

The equivalent circuit corresponding to this stage is shown in Fig. 2. During this interval, diode \(D₂\) and switch \(S\) are ON, whereas the diodes \(D₁\) and \(D\) are OFF. The input voltage source \(V₁\) appears in series with the inductors \(L₁\) and \(L₂\).

The corresponding state equations are as follows.

\[
\frac{di_{L₁}}{dt} = -\frac{V_{C₁}}{L₁} + \frac{V₁}{L₁} \tag{1}
\]

\[
\frac{di_{L₂}}{dt} = -\frac{V_{C₂}}{L₂} + \frac{V_{C₁}}{L₂} \tag{2}
\]

\[
\frac{dV_{C₁}}{dt} = \frac{i_{L₁}}{C₁} - \frac{i_{L₂}}{C₁} \tag{3}
\]

\[
\frac{dV_{C₂}}{dt} = \frac{i_{L₂}}{C₂} + \frac{V_{C₂}}{RC₂} \tag{4}
\]
C. Mode II:

The equivalent circuit corresponding to this stage is shown in Fig.3. During this time interval, the switch S and diode D_2 are turned OFF whereas diodes D_1 and D are ON. The inductor L_1 begins to discharge through the capacitor C_1 and L_2 discharge through C_2.

![Fig 3 circuit diagram for mode II](image)

The corresponding state equations are as follows.

\[
\frac{di_1}{dt} = \frac{-V_{C1}}{L_1} \tag{5}
\]

\[
\frac{di_2}{dt} = \frac{-V_{C2}}{L_2} \tag{6}
\]

\[
\frac{dV_{C1}}{dt} = \frac{i_1}{C_1} \tag{7}
\]

\[
\frac{dV_{C2}}{dt} = \frac{i_2 - V_{C2}}{C_2 R_{C2}} \tag{8}
\]

D. Mode III:

The equivalent circuit corresponding to this stage is shown in Fig.4. In this stage all the energy stored in C_2 was transferred to the load. With this, the diode D blocks and output capacitor C_2 feeds the load.

![Fig 4 circuit diagram for mode III](image)

The corresponding state equation is as follows.

\[
\frac{dV_{C2}}{dt} = \frac{-V_{C2}}{R_{C2}} \tag{9}
\]

III. VOLTAGE-MODE PULSE TRAIN CONTROL OF QUADRATIC BUCK CONVERTER IN DCM

Many control methods have widespread applications in the closed-loop control of conventional DC/DC buck converters but there are very few discussions on the close loop Control of quadratic buck converters. This may be due to the fact that a quadratic buck converter has more complex non-linearity than a single stage buck converter, which may result with an increased difficulty in designing controller for the quadratic converter. Voltage mode pulse train control QBC offers wide range of voltage regulation.
E. Voltage Mode Pulse Train Control:

Pulse train (PT) is a new control technique for switching DC-DC converters, which achieves output voltage regulation by generating a control pulse train constructed by two control pulses with different duty ratio. Pulse train technique has several advantages over conventional PWM control techniques, such as simplicity of design and implementation, and excellent transient performance. Furthermore, pulse train control switching DC-DC converters can operate in fixed switching frequency and achieve soft switching conveniently.

Pulse train control technique realizes the control of switching DC-DC converters by a pulse train constructing by two control pulses with different duty ratio, high power pulse $P_H$ and low power pulse $P_L$ is shown in fig6. At the beginning of each switching cycle, the output voltage $V_O$ is sensed and compared with the desired reference voltage $V_{ref}$. If $V_O$ is lower than $V_{ref}$, high power pulse $P_H$ will be generated as the control signal in this switching cycle. On the other hand, if $V_O$ is higher than $V_{ref}$, low power pulse $P_L$, which has a shorter on time compared with $P_H$, will be generated.
Obviously, the converter will deliver more power to the load during high power pulse cycle than during low power pulse cycle.

According to the principle described above, pulse train control method realizes output voltage regulation of switching DC-DC converter by adjusting the combination of control pulse of PH and PL. In steady state, control pulse train constructed by proper combination of PH and PL in sequential switching cycles is generated for the control of switching DC-DC converters. The pulse train in the sequential switching cycles composes a new repetition cycle. The on times of PH and PL are different, but the switching periods T are the same, therefore, the switching frequency of the converter is constant.

For voltage-mode pulse train control scheme, control pulses PH and PL have constant duty ratios DH and DL (DH > DL) respectively. In the case of high power pulse, the power switch is turned on at the beginning of each switching cycle, \( t = nT \), and turned off at time \( t = nT + D_HT \). It will keep off until the beginning of next switching cycle.

### Analysis of VMPT Control:

For the VMPT control quadratic buck converter operating in steady state, suppose the number of PH and PL in the pulse train repetition cycle is \( \mu_H \) and \( \mu_L \) respectively.

#### Average Switch current:

\[
I_{SW} = \frac{V_1 - V_2}{2L} \frac{D_H}{T} \tag{10}
\]

The energy drawn from the input during this pulse train repetition cycle is

\[
E_{inH} = \frac{V_1(V_1 - V_2)}{2L} \frac{D_H}{T} \tag{11}
\]

\[
E_{inL} = \frac{V_1(V_1 - V_2)}{2L} \frac{D_L}{T} \tag{12}
\]

\[
E_{in} = \mu_H E_{inH} + \mu_L E_{inL} \tag{13}
\]

\[
E_{in} = V_1 \frac{V_1 - V_2}{2L} T \left\{ \mu_H D_H^2 + \mu_L D_L^2 \right\} \tag{14}
\]

If the efficiency of the converter is \( \eta \), the output power \( P \) can be expressed as

\[
P(\mu_H + \mu_L)T = \eta E_{in} \tag{15}
\]

\[
P = \eta V_1 T \frac{V_1 - V_2}{2L(\mu_H + \mu_L)} \left\{ \mu_H D_H^2 + \mu_L D_L^2 \right\} \tag{16}
\]

The proportion between \( \mu_H \) and \( \mu_L \):

\[
\frac{\mu_H}{\mu_L} = \frac{1 - MD_L^2}{MD_H^2 - 1} \tag{17}
\]

where

\[
M = \eta V_1 T \frac{V_1 - V_2}{2LP} \tag{18}
\]

Suppose the input voltage of the VMPT control converter is constant, we can conclude that in each pulse train repetition cycle, when higher output power is required, the control strategy will generate more control pulse PH and less control pulse PL, and vice versa with lighter load. This conclusion can be well explained from the conversion of energy. If the load of buck converter increases, the converter needs to deliver more energy by using PH control pulses.

On the other hand, if the load of buck converter is constant, there will be more PH for the case of lower input voltage, and more PL for the case of higher input voltage. This can also be well explained from the conversion of energy. We know that when the input voltage of the converter increases, the energy drawn from the input power source in PH or PL switching cycle will increase. As the energy consume by load is constant, the number of PH in a pulse train repetition cycle will be fewer.
G. Effect of Varying Resistance:

<table>
<thead>
<tr>
<th>Resistance ranges(Ω)</th>
<th>Different periodicities and pulse repetition pattern</th>
<th>( \frac{p_H}{p_L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 10</td>
<td>Period-1(1P_H)</td>
<td></td>
</tr>
<tr>
<td>(13.8-15.1)</td>
<td>Period-2(1P_H-1P_L)</td>
<td>1</td>
</tr>
<tr>
<td>(18.7-19.1)</td>
<td>Period-3(1P_H-2P_L)</td>
<td>0.5</td>
</tr>
<tr>
<td>&gt;35</td>
<td>Period-1(1P_L)</td>
<td></td>
</tr>
</tbody>
</table>

H. Effect of Varying Input Voltage:

<table>
<thead>
<tr>
<th>Voltage ranges(v)</th>
<th>Different periodicities and pulse repetition pattern</th>
<th>( \frac{p_H}{p_L} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>&lt; 24</td>
<td>Period-1(1P_H)</td>
<td></td>
</tr>
<tr>
<td>(26-27.5)</td>
<td>Period-2(1P_H-1P_L)</td>
<td>1</td>
</tr>
<tr>
<td>(29.4-30.1)</td>
<td>Period-3(1P_H-2P_L)</td>
<td>0.5</td>
</tr>
<tr>
<td>&gt;36</td>
<td>Period-1(1P_L)</td>
<td></td>
</tr>
</tbody>
</table>

IV. SIMULATION RESULTS

The quadratic buck converter is simulated using MATLAB/SIMULINK with and without controller. When the Voltage mode Pulse train is implemented voltage regulation of the converter is achieved. The bifurcation analysis is done with the variation of load resistance.

A. Specifications:

<table>
<thead>
<tr>
<th>Circuit components</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Switching frequency</td>
<td>110 kHz</td>
</tr>
<tr>
<td>Input Voltage ( V_1 )</td>
<td>24V</td>
</tr>
<tr>
<td>Inductance ( L_1 )</td>
<td>10 µH</td>
</tr>
<tr>
<td>Inductance ( L_2 )</td>
<td>10 µH</td>
</tr>
<tr>
<td>Capacitance ( C_1 )</td>
<td>16 µF</td>
</tr>
<tr>
<td>Capacitance ( C_2 )</td>
<td>16 µF</td>
</tr>
<tr>
<td>Load resistance ( R )</td>
<td>10Ω</td>
</tr>
</tbody>
</table>

Table 1: Component values used in simulation
B. Simulation results without controller:

The quadratic buck converter is simulated without controller. The output voltage, output current, inductor current and capacitor voltages are analysed. The simulated waveform for the inductor current, capacitor voltage is shown in fig 7.

![Simulated waveform of inductor current and capacitor voltage](image)

**Fig 7** simulated waveform of inductor current and capacitor voltage

C. Simulations results with controller:

i)Varying load resistance:

The quadratic buck converter is analysed by implementing voltage mode pulse train control and the simulated results are shown below.

![Simulated waveform of inductor current and output voltage for R=8 Ω](image)

**Fig 8**. Simulated waveform of inductor current and output voltage for R=8 Ω

The waveforms for the quadratic buck converter with VMPT control for the load resistance of R=8Ω is shown in fig8. The control pulses within a pulse train repetition cycle are 1P_H in this case. The control pulses within a pulse train repetition cycle are contains only the lower width pulses (P_H) since the output voltage is lesser than that of the reference voltage.

The waveforms for the quadratic buck converter with VMPT control for the load resistance of R=14.5Ω is shown in fig 9.

![Simulated waveform of inductor current and output voltage for R=14.5Ω](image)

**Fig 9** Simulated waveform of inductor current and output voltage for R=14.5Ω

D. Simulations results with controller:

i)Varying load resistance:

The quadratic buck converter is analysed by implementing voltage mode pulse train control and the simulated results are shown below.
The waveforms for the quadratic buck converter with VMPT control for the load resistance of \( R=8 \Omega \) is shown in fig 8. The control pulses within a pulse train repetition cycle are \( 1P_{H} \) in this case. The control pulses within a pulse train repetition cycle are contains only the lower width pulses \( (P_{L}) \) since the output voltage is lesser than that of the reference voltage.

The waveforms for the quadratic buck converter with VMPT control for the load resistance of \( R=14.5 \Omega \) is shown in fig 9.

The same analysis has been carried out for varying input voltage where the resistance is kept constant. The waveforms for the quadratic buck converter with VMPT control for the input voltages of \( V_{in}=23V,29.5V \) are shown in Fig 10, Fig 11 respectively. The control pulses within a pulse train repetition cycle are \( 1P_{H},1P_{H}-2P_{L} \) in this case.

### V. CONCLUSION

The steady-state analysis of the quadratic buck converter in discontinuous conduction mode is performed. Using MATLAB simulation, the converter is analysed for the given set of specifications.
Pulse train is a new nonlinear control technique for switching DC-DC converters. The control strategy, operation and advantages of pulse train method are discussed. Based on comprehensive analysis and comparison on voltage-mode buck converter in DCM, some useful control regulations are obtained. The operation states of converter under different load resistance and input conditions are studied. Simulation and experimental results are presented to verify theory analysis results.

VI. ACKNOWLEDGMENT

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REFERENCES